Within-Person Variability Score-Based Causal Inference: A Two-Step Semiparametric Estimation for Joint Effects of Time-Varying Treatments

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Abstract

Behavioral science researchers have recently shown strong interest in disaggregating within- and between-person effects (stable traits) from longitudinal data. In this paper, we propose a method of within-person variability score-based causal inference for estimating joint effects of time-varying continuous treatments by effectively controlling for stable traits as time-invariant unobserved confounders. After conceptualizing stable trait factors and within-person variability scores, we introduce the proposed method, which consists of a two-step analysis. Within-person variability scores for each person, which are disaggregated from stable traits of that person, are first calculated using weights based on a best linear correlation preserving predictor through structural equation modeling. Causal parameters are then estimated via a potential outcome approach, either marginal structural models (MSMs) or structural nested mean models (SNMMs), using calculated within-person variability scores. We emphasize the use of SNMMs with G-estimation because of its doubly robust property to model errors. Through simulation and empirical application to data regarding sleep habits and mental health status from the Tokyo Teen Cohort study, we show that the proposed method can recover causal parameters well and that causal estimates might be severely biased if one does not properly account for stable traits.

Keywords: Longitudinal data, Observational study, Causal inference, Marginal structural model, Structural nested mean model

1 INTRODUCTION

Estimating the causal effects of (a sequence of) time-varying treatments/predictors on outcomes is a challenging issue in longitudinal observational studies, because researchers must account for time-varying and time-invariant confounders. For this analytic purpose, potential outcome approaches such as marginal structural models (MSMs; Robins, 1999; Robins, Hernán, & Brumback, 2000) have been widely used in epidemiology. Although actual applications have been relatively infrequent, structural nested models (SNMs; Robins, 1989, 1992) with G-estimation are in principle more suitable for handling violation of the usual assumptions of no unobserved confounders and sequential ignorability (Robins, 1999; Robins & Hernán, 2009; Vansteelandt & Joffe, 2014).

Parallel with such methodological development, behavioral science researchers have shown interest in inferring within-person relations in longitudinally observed variables, namely, how changes in one variable influence another for the same person. For example, Sampson, Laub, and Wimerhow (2006) used MSMs to investigate *within-individual causal effects* of being married or unmarried on crime behaviors. Investigations based on within-person relations might produce conclusions opposite to those based on between-person relations. For example, a person is more likely to have a heart attack during exercise (within-person relation), despite people who exercise more having a lower risk of heart attack (between-person relation; Curran & Bauer, 2011).

Statistical inference for disaggregating within- and between-person (or within- and between-group) effects has been a concern in behavioral sciences for more than half a century. However, recent methodological development and extensive discussion (Cole, Martin, & Steiger, 2005; Hamaker, 2012; Hamaker, Kuiper, & Grasman, 2015; Hoffman, 2014; Usami, Murayama, & Hamaker, 2019) have rapidly increased interest in this topic. In the

psychometrics literature, structural equation modeling (SEM)-based approaches have become a popular method for uncovering within-person relations. Among these approaches, applications of a random-intercept cross-lagged panel model (RI-CLPM; Hamaker et al., 2015), which include common factors called *stable trait factors* to control for stable individual differences over time, have rapidly increased, reaching more than 600 citations on Google as of June 2020. This model was originally proposed to uncover reciprocal relations among variables that arise at the within-person level (i.e., simultaneous investigations for the effects of a variable X on a variable Y, along with the effects of Y on X).

Despite its popularity and theoretical appeal, the concepts of within-person effects in the RI-CLPM have not been fully characterized in the causal inference literature. This might be partly because psychometricians have used the terms *stable traits* and *within-person relations* in ambiguous ways for different models (Usami, Murayama, & Hamaker, 2019), without providing clear mathematical definitions. For this reason, the RI-CLPM has not been contrasted with other methodologies, such as MSMs and SNMs. One potential advantage of the RI-CLPM as SEM is that it can easily include and estimate measurement errors in models under parametric assumptions. However, the RI-CLPM has a serious drawback: it requires correctly specified linear regressions to connect variables at the within-person level. The linearity that is typically assumed in path modeling and SEM has often been criticized in the causal inference literature (e.g., Hong, 2015).

In this paper, we propose a method of *within-person variability score*-based causal inference for estimating joint effects of time-varying (continuous) treatments/predictors by effectively controlling for stable traits (i.e., between-person differences) that can be viewed as time-invariant unobserved confounders. The proposed method is a two-step analysis. A within-person variability score for each person, which is disaggregated from the stable trait factor score of that person, is first calculated using weights based on a best linear

correlation preserving predictor through SEM. Causal parameters are then estimated by MSMs or SNMs, using calculated within-person variability scores. This approach is more flexible than is RI-CLPM when modeling how observed confounders are connected with outcomes and treatments/predictors. We particularly emphasize the utility of SNMs with G-estimation because of its doubly robust property to model errors. The proposed method can be considered as a semiparametric approach by synthesizing two traditions for factor analysis methods and SEM in psychometrics and a potential outcome approach (MSMs or SNMs) in epidemiology, and within-person variability scores can be applied to many other problems involving causal inference such as reciprocal effects and mediation effects.

The remainder of this paper is organized as follows. In Section 2 we review RI-CLPM, then conceptualize stable trait factors and within-person variability scores. After defining causal effects and identification conditions under within-person variability score-based inference in Section 3, we introduce the proposed methodology in Section 4. Through simulations described in Section 5 and an empirical application presented in Section 6, we show that the proposed method can well recover causal parameters, and that causal estimates might be severely biased if stable traits are not properly considered. The final section gives some concluding remarks and discusses our future research agenda.

2 CONCEPTUALIZING STABLE TRAIT FACTORS AND WITHIN-PERSON VARIABILITY SCORES

2.1 RI-CLPM

In psychology, *traits* were originally considered as personality characteristics that are stable over time and in different situations. To express such latent constructs, common factors are

explicitly included in psychometric models. In the context of the RI-CLPM, such common factors are called *stable trait factors*.

The original motivation for the RI-CLPM was to infer reciprocal (rather than unidirectional) relations between two variables, and the model does not include observed confounders. For explanatory purposes, here we assume a longitudinal study that applies RI-CLPM to investigate effects of continuous treatments A on outcome Y that occur at the within-person level in the presence of observed (time-varying) confounders L.

Suppose that measurements are collected at fixed time points $t_0, t_1, \ldots, t_K$. Let A_{ik} denote a continuous treatment/predictor at time t_k $(k = 0, \ldots, K-1)$ for person i, and let L_{ik} denote observed confounders measured at that time for person i. Further, Y_{ik} is the outcome measured at time t_k $(k = 0, \ldots, K)$ for person i, and is part of time-varying confounders L_{ik}. Suppose time-varying confounders has three characteristics: it is independently associated with future outcomes $Y_{ik'}$, it predicts subsequent levels of treatment as well as future confounders, and it is affected by an earlier treatment and confounders (Vansteelandt & Joffe, 2014). In this paper, we assume a single covariate, which is concurrently measured with the outcome at each time point and is measured before the assignment/predictor level is determined for each person. Thus, we presume the variables are ordered as $L_0, A_0, L_1, A_1, \ldots, L_{K-1}, A_{K-1}, L_K$.

In the RI-CLPM, observations are first modeled as

$$Y_{ik} = \mu_k^{(Y)} + I_i^{(Y)} + Y_{ik}^*, \quad A_{ik} = \mu_k^{(A)} + I_i^{(A)} + A_{ik}^*, \quad L_{ik} = \mu_k^{(L)} + I_i^{(L)} + L_{ik}^*, \tag{1}$$

where $\mu_k^{(Y)}$, $\mu_k^{(A)}$, and $\mu_k^{(L)}$ are the temporal group means at time t_k, and $I_i^{(Y)}$, $I_i^{(A)}$ and $I_i^{(L)}$ are time-invariant stable trait factors that represent a person's trait-like deviations from the temporal group means. Stable trait factor means are fixed to 0 while their (co)variances are estimated. By accounting for stable trait factors, Y_{ik}^*, A_{ik}^*, and L_{ik}^* represent temporal

deviations from the expected score for person i at time t_k (i.e., $\mu_k^{(Y)} + I_i^{(Y)}$, $\mu_k^{(A)} + I_i^{(A)}$, and $\mu_k^{(L)} + I_i^{(L)}$). Means of these deviations are 0. Time series Y_{ik}^*, A_{ik}^*, and L_{ik}^* express within-person variations that are independent from stable trait factors (between-person differences) during the study.

Considering the order of variable observations, the RI-CLPM might model temporal deviations using first-order autoregression at $k \geq 1$ as

$$\begin{aligned} Y_{ik}^* &= \alpha_k^{(Y)} Y_{i(k-1)}^* + \beta_k^{(Y)} A_{i(k-1)}^* + \gamma_k^{(Y)} L_{i(k-1)}^* + d_{ik}^{(Y)} \\ A_{ik}^* &= \alpha_k^{(A)} Y_{ik}^* + \beta_k^{(A)} A_{i(k-1)}^* + \gamma_k^{(A)} L_{ik}^* + d_{ik}^{(A)} \\ L_{ik}^* &= \alpha_k^{(L)} Y_{i(k-1)}^* + \beta_k^{(L)} A_{i(k-1)}^* + \gamma_k^{(L)} L_{i(k-1)}^* + d_{ik}^{(L)}, \end{aligned} \tag{2}$$

where $\beta_k^{(Y)}$ indicates a cross-lagged parameter that is key to inferring within-person treatment effects on the outcome. The initial deviations (Y_{i0}^*, A_{i0}^* and L_{i0}^*) are modeled as exogeneous variables, and their variances and covariances are estimated. Residual d is usually assumed to follow a multivariate normal distribution. Parameters are typically estimated via an SEM-based maximum likelihood (ML) method, which uses the model-implied mean and covariance structures. Assuming sufficient sample size, correct model specification, and no excess multivariate kurtosis, ML provides estimates and SEs that are asymptotically unbiased, efficient, and consistent (Bollen, 1989).

Note that multilevel modeling (including lagged effects) is another popular approach to disaggregating between- and within-person effects of treatments/predictors (e.g., Curran & Bauer, 2011; Sampson et al., 2006; Wang & Maxwell, 2015). Variable centering, which is a common and appropriate option for dissociating effects at different levels, is an important issue in multilevel modeling (Asparouhov & Muthén, 2018; Enders & Tofighi, 2007). It can be shown that including stable trait factors as in the RI-CLPM is equivalent to assuming random intercepts in the multilevel model, because the combined form of Equations (1)

and (2) (say, for the outcome model) becomes

$$Y_{ik} - \mu_k^{(Y)} = I_i^{(Y)} + \alpha_k^{(Y)} Y_{i(k-1)}^* + \beta_k^{(Y)} A_{i(k-1)}^* + \gamma_k^{(Y)} L_{i(k-1)}^* + d_{ik}^{(Y)}. \quad (3)$$

This equation can be viewed as a level-1 (unit-level) equation of a random intercept model with lagged effects, indicating that stable trait factor $I_i^{(Y)}$ can be interpreted as a random intercept. This comparison suggests a similarity between the RI-CLPM and the multilevel model. However, unlike RI-CLPM, predictor centering in a multilevel model is usually conducted by observed person-specific means (e.g., $A_{i(k-1)} - \bar{A}_i$) rather than stable trait factors ($A_{i(k-1)} - I_i^{(A)}$) (for exceptions using latent mean centering, see Asparouhov & Muthén, 2018). One drawback of using observed person-specific means is that they include components of temporal deviations (within-person variations), and thus fail to perfectly disaggregate true between- and within-person effects. This problem becomes critical when K is small. In addition, subtracting observed person-specific means leads to a rank-deficient matrix for each variable, causing the statistical analysis to become intractable if one is interested in estimating the effects of all past treatments on the outcome at the last time point (t_K). This problem with the use of observed person-specific (or cluster-specific) means to express cluster effects is also widely recognized in other applications of the multilevel model (e.g., Asparouhov & Muthén, 2018; Lüdtke et al., 2008; Usami, 2017).

2.2 Definition of Stable Trait Factors and Within-person Variability Scores

Psychometricians have used the term *(stable) traits* and *within-person relations* in ambiguous ways for various SEM-based longitudinal models, despite mathematical and interpretative differences existing (e.g., Hamaker, 2015; Usami, Murayama, & Hamaker, 2019). For example, common factors included in autoregressive latent trajectory models (Bollen &

Curran, 2004) and individual-specific effects that are sometimes included in longitudinal panel models of econometrics (e.g., the ARMA model) commonly have *both* direct and indirect effects on observations, while the stable trait factors in the RI-CLPM have only direct effects that are invariant over time (i.e., Equation 1).

Inspired by the formulation of the RI-CLPM, in this paper a *stable trait factor* as a between-person difference for person i (say, for Y) is defined as the difference between expected values of observation (true score) of this person at time t_k ($\mu_{ik}^{(Y)}$) and temporal group mean at time t_k ($\mu_k^{(Y)}$) that are *invariant* over time:

$$I_i^{(Y)} = \mu_{ik}^{(Y)} - \mu_k^{(Y)}, \tag{4}$$

for $k = 0, \ldots, K$, $-\infty < \mu_{ik}^{(Y)} < \infty$, and $-\infty < \mu_k^{(Y)} < \infty$. Note that $E(I_i^{(Y)}) = E(\mu_{ik}^{(Y)} - \mu_k^{(Y)}) = \mu_k^{(Y)} - \mu_k^{(Y)} = 0$.

Next, *within-person variability score* Y_{ik}^* as the temporal deviation of person i at time t_k is defined as the difference between an observation and its expected value as

$$Y_{ik}^* = Y_{ik} - \mu_{ik}^{(Y)} = Y_{ik} - (\mu_k^{(Y)} + I_i^{(Y)}), \tag{5}$$

assuming that $E(Y_{ik}^*)$=0 and $Cov(\mu_{ik}^{(Y)}, Y_{ik}^*) = 0$ (i.e., expected values of observations and within-person variability scores are uncorrelated). Note that stable trait factors and within-person variability scores are uncorrelated, because

$$Cov(I_i^{(Y)}, Y_{ik}^*) = E[(\mu_{ik}^{(Y)} - \mu_k^{(Y)})Y_{ik}^*] = E(\mu_{ik}^{(Y)}Y_{ik}^*) - \mu_k^{(Y)}E(Y_{ik}^*) = 0. \tag{6}$$

Thus, observation variances at time t_k can be expressed as the sum of variances of stable trait factor scores and within-person variability scores. This means the time series for Y_{ik}^* has the following covariance structure:

$$Cov(Y_{ik}^*, Y_{ik'}^*) = Cov(Y_{ik}, Y_{ik'}) - Var(I_i^{(Y)}). \tag{7}$$

3 CAUSAL EFFECTS UNDER WITHIN-PERSON VARIABILITY SCORE-BASED INFERENCE

Below, we use overbars $\bar{Y}_k = \{Y_0, Y_1, \ldots, Y_k\}$ to denote the history of Y through t_k and underbars $\underline{Y}_k = \{Y_k, \ldots, Y_K\}$ to denote the future of this variable. Let $Y_{ik}^{\bar{A}_{i(k-1)}}$ denote the outcome that would be seen at time t_k for person i were this person to receive treatment history $\bar{A}_{i(k-1)} = \{A_{i0}, \ldots, A_{i(k-1)}\}$ through time t_{k-1}. This variable is a potential outcome, which we connect to observations by the consistency assumption

$$Y_{ik} = Y_{ik}^{\bar{a}_{i(k-1)}} \tag{8}$$

if $\bar{A}_{i(k-1)} = \bar{a}_{i(k-1)}$. Otherwise, $Y_{ik}^{\bar{a}_{i(k-1)}}$ is counterfactual. The standard assumption of no unobserved confounders or sequential ignorability indicates that

$$\underline{Y}_{ik}^{\bar{a}_{i(k-2)},0} \perp\!\!\!\perp A_{i(k-1)} \mid \bar{L}_{i(k-1)} = \bar{l}_{i(k-1)}, \bar{A}_{i(k-2)} = \bar{a}_{i(k-2)} \tag{9}$$

for $k = 1, \ldots, K$. Here, $(\bar{a}_{i(k-2)}, 0)$ is the counterfactual history, that is, the history that agrees with $\bar{a}_{i(k-2)}$ through time t_{k-2} and is 0 thereafter.

Under the assumption of the stable unit treatment value assumption (e.g., Hong, 2015), the average causal effect when a continuous treatment/predictor increases one unit from the reference value $a^r_{i(k-1)}$ at time t_{k-1} can be defined using differences in average potential outcomes given information on confounders and treatment history as

$$\begin{aligned}
&E(Y_{ik}^{\bar{a}_{i(k-2)},a^r_{i(k-1)}+1}) - E(Y_{ik}^{\bar{a}_{i(k-2)},a^r_{i(k-1)}}) \\
=&E(Y_{ik}|\bar{L}_{i(k-1)} = \bar{l}_{i(k-1)}, \bar{A}_{i(k-2)} = \bar{a}_{i(k-2)}, A_{i(k-1)} = a^r_{i(k-1)} + 1) \\
&- E(Y_{ik}|\bar{L}_{i(k-1)} = \bar{l}_{i(k-1)}, \bar{A}_{i(k-2)} = \bar{a}_{i(k-2)}, A_{i(k-1)} = a^r_{i(k-1)}).
\end{aligned} \tag{10}$$

Average joint effects of past treatments/predictors can be defined in the same manner.

In within-person variability score-based causal inference that explicitly controls for stable trait factors, the assumption of sequential ignorability can be modified as an argument regarding within-person variability scores as

$$\underline{Y}^{*\,\bar{a}^*_{i(k-2)},0}_{ik} \perp\!\!\!\perp A^*_{i(k-1)} | \bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-2)} = \bar{a}^*_{i(k-2)}. \tag{11}$$

Let $Y^{*\,\bar{a}^*_{i(k-1)}}_{ik}$ denote the potential outcome based on the within-person variability score at time t_k for person i with treatments/predictors $\bar{a}^*_{i(k-1)} = (a^*_{i0}, \ldots, a^*_{i(k-1)})$ through time t_{k-1}. $a^*_i = 0$ now indicates the average amount of treatments for person i.

From Equations (5) and (6), the above assumption is mathematically equivalent to the conditional independence regarding potential outcomes that are connected to the observations ($\underline{Y}^{\bar{a}^*_{i(k-2)},0}_{ik}$), rather than within-person variability scores:

$$\underline{Y}^{\bar{a}^*_{i(k-2)},0}_{ik} \perp\!\!\!\perp A^*_{i(k-1)} | \bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-2)} = \bar{a}^*_{i(k-2)}. \tag{12}$$

Thus, in within-person variability score-based causal inference, the average causal effect when within-person variability scores of treatments/predictors increase one unit from the reference value a^{*r}_{k-1} at time t_{k-1} can be defined using differences in average potential outcomes $E(Y^{\bar{a}^*_{i(k-1)}}_{ik})$ as

$$\begin{aligned}
&E(Y_{ik}{}^{\bar{a}^*_{i(k-2)},a^{*r}_{i(k-1)}+1}) - E(Y_{ik}{}^{\bar{a}^*_{i(k-2)},a^{*r}_{i(k-1)}}) \\
=&E(Y_{ik}|\bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-2)} = \bar{a}^*_{i(k-2)}, A^*_{i(k-1)} = a^{*r}_{i(k-1)} + 1) \\
&- E(Y_{ik}|\bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-2)} = \bar{a}^*_{i(k-2)}, A^*_{i(k-1)} = a^{*r}_{i(k-1)}),
\end{aligned} \tag{13}$$

or, equivalently, using the difference of average potential outcomes connected to within-person variability scores $E(Y^{*\,\bar{a}^*_{i(k-1)}}_{ik})$.

Stable traits in outcomes ($I^{(Y)}$) can be considered as a cause of Y_k at time t_k, and at the same time they are associated with (stable traits of) treatments/predictors. Similarly, $I^{(L)}$

and $I^{(A)}$ can be considered as a cause of observed confounders L_k and treatments/predictors A_k, respectively, and they are associated with (stable traits of) treatments/predictors. In that sense, stable trait factors can be viewed as time-invariant unobserved confounders (Usami, Murayama, & Hamaker, 2019), and the difference between Equations (10) and (13) can be viewed as the kind of confounders that are controlled for. This implies that within-person variability score-based causal inference might be advantageous when estimating causal parameters, because it poses less risk of violating the identifiability assumption of sequential ignorability.

If potential outcomes at time t_k depend on only the previous state of treatments/predictors and observed confounders (with no interaction effects) at the within-person level, and if treatments/predictors and confounders are linearly related to potential outcomes, $\beta_k^{(Y)}$ in the RI-CLPM (Equation 2) represents the causal effects of a treatment or predictor at time t_{k-1} on an outcome at time t_k, because Equation (13) becomes

$$[\alpha_k^{(Y)} Y_{i(k-1)}^* + \beta_k^{(Y)}(A_{i(k-1)}^* + 1) + \gamma_k^{(Y)} L_{i(k-1)}^*] - [\alpha_k^{(Y)} Y_{i(k-1)}^* + \beta_k^{(Y)} A_{i(k-1)}^* + \gamma_k^{(Y)} L_{i(k-1)}^*] = \beta_k^{(Y)}. \tag{14}$$

However, the RI-CLPM as SEM requires a strong assumption of linear regression to connect variables that are correctly specified (Equation 2), which has often been criticized in the causal reference literature (e.g., Hong, 2015). Ensuring a correct specification is very challenging in longitudinal designs.

4 PROPOSED METHODOLOGY

We are now ready to introduce a method of within-person variability score-based causal inference for estimating joint effects of time-varying continuous treatments. The proposed method consists of a two-step analysis. Within-person variability scores are first calculated

using weights through SEM. Causal parameters are then estimated by MSMs or SNMs, using calculated within-person variability scores. This approach is more flexible than the RI-CLPM when modeling how observed confounders are connected to outcomes and treatments/predictors. Before explaining the proposed methodology, we briefly discuss the motivation for adopting a two-step method, rather than simultaneously estimating stable trait factors (or within-person scores) and causal parameters.

In general, partial misspecification in measurements and/or structural models is known to cause large biases in estimates of model parameters. In the present context, when a simultaneous estimation method like the RI-CLPM is used, misspecification in the structural models at the within-person level (i.e., Equation 2) may greatly affect parameter estimates in the measurement model (i.e., Equation 1: (co)variances of stable factors and within-person variability scores), and vice versa.

To avoid such confounding, in the SEM context Anderson and Gerbing (1988) proposed a two-step procedure that first confirms the measurement model with a saturated model, so that structural relations have no impact on the measurement model. Then, using an appropriate measurement model, the substantive structural relations model of interest is added (Hoshino & Bentler, 2013). Applications of similar multistep estimation procedures can be seen for diverse classes of latent variable models (Bakk & Kuha, 2017; Croon, 2002; Skrondal & Laake, 2001; Vermunt, 2010).

Another potential advantage of two-step estimation is its feasibility. Parameters in measurement models can be estimated through various software packages for SEM, including Amos, SAS PROC CALIS, R packages (sem, lavaan, OpenMx), LISREL, EQS, and Mplus. MSMs and SNMs can be straightforwardly applied just by using calculated within-person variability scores instead of observations.

Two-step estimation is also advantageous because it poses less risk of improper solu-

tions. This problem is often encountered when applying the RI-CLPM because of negative variance parameters and a singular approximate Hessian matrix for stable trait factor variance–covariance (e.g., Usami, Todo, & Murayama, 2019), which is likely caused by misspecifications in linear regressions (i.e., the structural model). We will separately model stable trait factors for each variable (Y, A, and L) without influence from specified structural models, thus minimizing the risk of improper solutions.

4.1 Step 1: Estimation of Stable Trait Factor Scores and Within-Person Variability Scores

The first step of our method is divided into two sub-steps: (1) specification of the measurement models and parameter estimation and (2) prediction of within-person variability scores.

4.1.1 Specification of the measurement models and parameter estimation

Equation (1), which decomposes observations into a stable trait factor (between-person difference) and deviations (within-person variability scores), can be viewed as a factor analysis model that includes a single common factor I (whose factor loadings are all one) and a unique factor as deviations. In vector notation, Equation (1) for outcome Y becomes

$$Y_i = \mu^{(Y)} + I_i^{(Y)} 1_{K+1} + Y_i^*, \tag{15}$$

where $\mu^{(Y)}$ is a $(K+1) \times 1$ mean vector, $E(I_i^{(Y)}) = 0$, $Var(I_i^{(Y)}) = \phi^2_{(Y)}$, $E(Y_i^*) = 0$, and $Cov(I_i^{(Y)}, Y_i^*) = 0$. We denote as $\Psi_{(Y)}$ a $(K+1) \times (K+1)$ variance–covariance matrix of unique factors (within-person variability) scores. This implies that the variance–covariance matrix of Y (denoted as $\Sigma_{(Y)}$) is of the form $\Sigma_{(Y)} = \phi^2_{(Y)} 1_{K+1} 1^t_{K+1} + \Psi_{(Y)}$.

Unlike the standard factor analysis model, $\Psi_{(Y)}$ has a dependence structure and is not diagonal, so some structure, such as compound symmetry, a Toeplitz structure, or a first-order autoregressive (AR) structure, must be specified in $\Psi_{(Y)}$ for model identification. When the model is correctly specified, consistent estimators for $\mu^{(Y)}$, $\phi^2_{(Y)}$, and $\Psi_{(Y)}$ can be obtained by MLE in SEM (Jöreskog & Lawley, 1968).

In SEM, missing values can be easily handled by full information maximum likelihood (Enders & Bandalos, 2001) with the assumption of missing at random (MAR; Rubin, 1976). If data are suspected to be missing not at random, then appropriate sensitivity analyses and/or multiple imputation should be considered (Resseguier, Giorgi, & Paoletti, 2011).

Another advantage of SEM is that validity of the specified model can be diagnosed via multiple model-fit indices, along with model comparisons using information criteria such as AIC and BIC. In this paper, we use three current major indices (e.g., Hu & Bentler, 1999; Kline, 2016): the comparative fit index (CFI), (2) the root-mean square error of approximation (RMSEA) and (3) the standardized root-mean square residual (SRMR).

Similarly, we also set measurement models for treatments/predictors A and observed confounders L separately in this sub-step, then estimate parameters for mean vectors ($\mu^{(A)}$ and $\mu^{(L)}$), stable trait factor variances ($\phi^2_{(A)}$ and $\phi^2_{(L)}$), and unique factors variances $\Psi_{(A)}$ and $\Psi_{(L)}$.

4.1.2 Predicting within-person variability scores

Let $X_i = (Y_i, A_i, L_i)^t$ and $X_i^* = (Y_i^*, A_i^*, L_i^*)^t$ be vectors of observation and within-person variability scores, respectively, and let $\mu = (\mu^{(Y)}, \mu^{(A)}, \mu^{(L)})^t$ be a mean vector. Also let Σ and Ψ be covariance matrices for observations X_i and within-person variability scores X_i^*.

We consider linear prediction of within-person variability scores $\hat{X}_i^*$ under the condition that Σ and Ψ are known. Consider a $(3K+1)$ by $(3K+1)$ weight matrix W that provides

within-person variability scores from observations as

$$\hat{X}_i^* = W^t(X_i - \mu), \tag{16}$$

satisfying the relation

$$E(\hat{X}_i^* \hat{X}_i^{*t}) = W^t E[(X_i - \mu)(X_i - \mu)^t]W = W^t \Sigma W = \Psi. \tag{17}$$

Unlike standard applications of factor analysis, we are interested in predicting within-person variability (unique factor) scores, rather than stable trait factor (common factor) scores. However, the current problem of determining weights W shares the similar motivation of predicting factor scores. In the factor analysis literature, a predictor that preserves the covariance structure of common factors has been developed as a linear correlation preserving predictor (Anderson & Rubin, 1956; Green, 1969; ten Berge, Krijinen, Wansbeek, & Shapiro, 1999).

With this point in mind, W that can provide the best linear predictor of $\hat{X}_i^*$ minimizing the risk function, defined as the trace of a residual covariance matrix (i.e., mean squared error MSE($\hat{X}_i^*$)=$E[(\hat{X}_i^* - X_i^*)^t(\hat{X}_i^* - X_i^*)]$), which also satisfies the relation in Equation (17), can be obtained by utilizing singular value decomposition as

$$W^t = \Psi^{1/2}(\Psi^{3/2}\Sigma^{-1}\Psi^{3/2})^{-1/2}\Psi^{3/2}\Sigma^{-1}. \tag{18}$$

Here, for a positive (semi)definite matrix C, we denote as $C^{1/2}$ the positive (semi)definite matrix such that its square equals C. Matrices $C^{-1/2}$ and $C^{3/2}$ are the inverse (if it exists) and the third power of $C^{1/2}$, respectively. A derivation of W is provided in the Online Supplemental Material.

We use the sample means $\bar{X}$ and covariance matrix S of X as estimators of μ and Σ. As implied from the relation in Equation (7), we use estimated stable trait factor variances

to estimate Ψ as

$$\hat{\Psi} = S - \hat{\Phi}^{+}, \tag{19}$$

where $\hat{\Phi}^{+}$ consists of estimated stable trait factor (co)variances. In the simple case where the initial observation of Y (Y_0) is missing and the number of measurements equals K for each variable, $\hat{\Phi}^{+}$ becomes

$$\hat{\Phi}^{+} = \hat{\Phi} \otimes 1_K 1_K^t = \begin{pmatrix} \hat{\phi}^2_{(Y)} & \hat{\phi}_{(Y,A)} & \hat{\phi}_{(Y,L)} \\ \hat{\phi}_{(Y,A)} & \hat{\phi}^2_{(A)} & \hat{\phi}_{(A,L)} \\ \hat{\phi}_{(Y,L)} & \hat{\phi}_{(A,L)} & \hat{\phi}^2_{(L)} \end{pmatrix} \otimes 1_K 1_K^t, \tag{20}$$

where $\hat{\Phi}$ is an estimator of a 3×3 stable trait factor covariance matrix Φ. Because stable trait factor covariances are not estimated in the previous sub-step, we use covariances between calculated linear correlation preserving predictors for variables. For example, this predictor for Y can be expressed as

$$\hat{I}_i^{(Y)} = \frac{\hat{\phi}_{(Y)}}{\sqrt{1_{K+1}^t \hat{\Sigma}^{-1}_{(Y)} 1_{K+1}}} 1_{K+1}^t \hat{\Sigma}^{-1}_{(Y)} (Y_i - \bar{Y}). \tag{21}$$

$\hat{I}_i^{(A)}$ and $\hat{I}_i^{(L)}$ can be calculated in the same manner, whereby we obtain $\hat{\phi}_{(Y,A)} = Cov(\hat{I}_i^{(Y)}, \hat{I}_i^{(A)})$, $\hat{\phi}_{(Y,L)} = Cov(\hat{I}_i^{(Y)}, \hat{I}_i^{(L)})$ and $\hat{\phi}_{(A,L)} = Cov(\hat{I}_i^{(A)}, \hat{I}_i^{(L)})$. Predictors $\hat{I}_i = (\hat{I}_i^{(Y)}, \hat{I}_i^{(A)}, \hat{I}_i^{(L)})^t$ satisfy the relation $E(\hat{I}_i \hat{I}_i^t) = \Phi$ if the model is correctly specified in the previous sub-step. From Equations (16) and (18)–(20), we can thus obtain $\hat{X}_i^*$ without specifying the structural models that connect within-person variability scores from different variables (Y, A, and L), successfully maintaining independence from the next step.

4.2 Applying MSMs and SNMMs

The second step of the proposed method is straightforward, because we just need to apply MSMs or SNMs using calculated within-person variability scores. The following briefly

describes how MSMs and SNMs are implemented. Though they are actually calculated in the previous step, for simplicity of notation we omit the hat symbol to express within-person variability scores as Y_{ik}^{*}, A_{ik}^{*}, and L_{ik}^{*}.

Robins and co-workers developed SNMs with G-estimation (Robins, 1989; Robins, Blevins, Ritter, & Wulfsohn, 1992) and MSMs with an inverse probability weights (IPW) estimator (Robins, 1999; Robins, Hernán & Brumback, 2000). These methods have been extended to treat clustered outcomes (e.g., Brumback, He, Prasad, Freeman, & Rheingans, 2014; He, Stephens-Shields, & Joffe, 2015, 2019), though causal effects based on within-person variability scores (or, latent mean centering of all variables) have not been investigated in this area.

MSMs are advantageous in that they can be easily understood and fit with standard, off-the-shelf software that allows for weights (e.g., He, Stephens-Shields, & Joffe, 2019; Vansteelandt & Joffe, 2014). However, it is well-known that MSMs can be highly sensitive to misspecification of the treatment assignment model, even when there is a moderate number of time points (e.g., Hong, 2015; Lefebvre, Delaney, & Platt, 2008). Imai and Ratkovic (2015) proposed a covariate balancing propensity score methodology for robust IPW estimation.

Because of the doubly robust property of G-estimators, SNMs are a better approach for handling violation of the usual assumptions of no unmeasured confounders or sequential ignorability (Vansteelandt & Joffe, 2014). In addition, SNMs can allow direct modeling of the interactions and moderation effects of treatments/predictors A with observed confounders L. Another advantage of SNMs is that the variance of locally efficient IPW estimators in MSMs exceeds that of G-estimators in SNMs, unless A and L are independent. We therefore emphasize the utility of SNMs in this paper. Because we are now interested in evaluating the joint effects of treatments on the mean of an outcome, rather than those on

the entire distribution of the outcome, we apply structural nested mean models (SNMMs; Robins, 1994).

Note that potential disadvantages of SNMs are their limited utility for G-estimation when applying logistic SNMs and their limited availability of off-the-shelf software. Regarding the latter point, Wallace, Moodie, and Stephens (2017) recently developed an R package for G-estimation of SNMMs. The Online Supplemental Materials provide the R code that we used in the simulations.

4.2.1 MSMs using within-person variability scores

MSMs are typically applied to evaluate the sequence of treatment effects on the outcome, which is measured only at the end of a fixed follow-up period (K). For generality of discussion, however, here we assume that the outcome is measured each time and that the primary interest is evaluation of the sequence of past treatment effects on outcomes at each time point.

MSMs consider the marginal mean of potential outcomes $E(Y_{ik}^{\bar{a}_{i(k-1)}})$ at time t_k with treatment history $\bar{A}_{i(k-1)} = \bar{a}_{i(k-1)}$. In the current context, we consider potential outcomes at the within-person level, namely, $E(Y_{ik}^{*\bar{A}^*_{i(k-1)}})$ with treatment history $\bar{A}^*_{i(k-1)} = \bar{a}^*_{i(k-1)}$. Thus, $E(Y_{ik}^{*\bar{A}^*_{i(k-1)}})$ might take the form

$$E(Y_{ik}^{*\bar{A}^*_{i(k-1)}}) = \iota_0 + \sum_{t=1}^{k} \iota_t A^*_{i(k-1)}. \tag{22}$$

Parameters $\iota = (\iota_0, \ldots, \iota_K)^t$ can be estimated by fitting a weighted conditional model with an IPW estimator. In this example, the conditional model might take the form $E(Y^*_{ik}|\bar{A}^*_{i(k-1)} = \bar{a}^*_{i(k-1)}) = \iota_0 + \sum_{t=1}^{k} \iota_t a^*_{i(k-1)}$. One useful option for calculating weights is to use stabilized weights w_{ik} for person i at time point t_k (Hernán, Brumback, & Robins,

2002) as

$$w_{ik} = \prod_{t=1}^{k-1} \frac{f(A^*_{it}|A^*_{i(t-1)})}{f(A^*_{it}|A^*_{i(t-1)}, L^*_{it})}, \tag{23}$$

where $f(A^*_{it}|A^*_{i(t-1)}, L^*_{it}) > 0$ for all A^*_{it}, if $f(A^*_{i(t-1)}, L^*_{it}) \neq 0$ (the positivity assumption). Parameters will be biased if $f(A^*_{it}|A^*_{i(t-1)}, L^*_{it})$ is misspecified, but misspecification of $f(A^*_{it}|A^*_{i(t-1)})$ does not result in bias.

4.2.2 SNMMs using within-person variability scores

SNMMs simulate the sequential removal of an amount (*blip*) of treatment at t_{k-1} on subsequent average outcomes, after having removed the effects of all subsequent treatments. SNMMs then model the effect of a blip in treatment at t_{k-1} on the subsequent outcome means while holding all future treatments fixed at a reference level 0 (Vansteelandt & Joffe, 2014); in other words, the level that is equal to expected scores of a person in the current context. SNMMs parameterize contrasts of $\underline{Y}^{*\bar{a}^*_{i(k-1)},0}_{ik}$ and $\underline{Y}^{*\bar{a}_{i(k-2)},0}_{ik}$ conditionally on treatments/predictors and covariate histories through $t_{(k-1)}$ as

$$g[E(\underline{Y}^{*\bar{a}^*_{i(k-1)},0}_{ik}|\bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-1)} = \bar{a}^*_{i(k-1)})] - g[E(\underline{Y}^{*\bar{a}^*_{i(k-2)},0}_{ik}|\bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-1)} = \bar{a}^*_{i(k-1)})]$$
$$= h_k(\bar{l}^*_{i(k-1)}, \bar{a}^*_{i(k-1)}; \tau) \tag{24}$$

for each $k = 1, \ldots, K$, where $g(\cdot)$ is a known link function, and $h_k(\bar{l}^*_{i(k-1)}, \bar{a}^*_{i(k-1)}; \tau)$ is a known $(K - k + 1)$-dimensional function, smooth in finite-dimensional parameter τ.

In the following empirical applications using the data of $K = 2$, a linear SNMM using the identity link $g(x) = x$ is given by

$$\begin{aligned}
&E(Y^{*a_{i0},a_{i1}}_{i2} - Y^{*a_{i0},0}_{i2}|\bar{L}^*_{i1} = \bar{l}^*_{i1}, \bar{A}^*_{i1} = \bar{a}^*_{i1}) = (\tau_0 + \tau_1 l^*_{i1})a^*_{i1} \\
&E(Y^{*a_{i0},0}_{i2} - Y^{*0,0}_{i2}|L^*_{i0} = l^*_{i0}, A^*_{i0} = a^*_{i0}) = (\tau_2 + \tau_3 l^*_{i0})a^*_{i0} \\
&E(Y^{*a_{i0},0}_{i1} - Y^{*0,0}_{i1}|L^*_{i0} = l^*_{i0}, A^*_{i0} = a^*_{i0}) = (\tau_4 + \tau_5 l^*_{i0})a^*_{i0}.
\end{aligned} \tag{25}$$

Here, the first equation models the effect of A^*_{i1} on Y^*_{i2}, the second models the effect of A^*_{i0} on Y^*_{i2}, and the third models the effect of A^*_{i0} on Y^*_{i1}.

SNMMs consider a transformation $U^*_{im}(\tau)$ of $\underline{Y}^*_{ik}$, the mean value of which is equal to the mean that would be observed if treatment were stopped from time t_{k-1} onward, in the sense that

$$E(U^*_{i(k-1)}(\tau)|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)} = \bar{a}^*_{i(k-2)}, A^*_{i(k-1)}) = E(\underline{Y}^{*\bar{a}^*_{i(k-2)},0}_{ik}|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)} = \bar{a}^*_{i(k-2)}, A^*_{i(k-1)}) \tag{26}$$

for $k = 1, \ldots, K$ (Vansteelandt & Joffe, 2014). Here, $U^*_{i(k-1)}(\tau)$ is a vector with components $Y_{im} - \sum_{l=k-1}^{m-1} h^*_{l,m}(\bar{L}^*_{il}, \bar{A}^*_{il}; \tau)$ for $m = k, \ldots, K$ if $g(\cdot)$ is the identity link. For instance, in the above example of $K = 2$,

$$\begin{aligned} U^*_{i1}(\tau) &= Y^*_{i2} - (\tau_0 + \tau_1 L^*_{i1})A^*_{i1} \\ U^*_{i0}(\tau) &= (Y^*_{i1} - (\tau_4 + \tau_5 L^*_{i0})A^*_{i0}, Y^*_2 - (\tau_0 + \tau_1 L^*_{i1})A^*_{i1} - (\tau_2 + \tau_3 L^*_{i0})A^*_{i0})^t. \end{aligned} \tag{27}$$

The assumption of sequential ignorability (Equations 11 and 12) together with identity (Equation 26) imply that

$$E(U^*_{i(k-1)}(\tau^*)|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-1)}) = E(U^*_{i(k-1)}(\tau^*)|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)}) \tag{28}$$

for $k = 1, \ldots, K$. The parameters τ can therefore be estimated by solving the estimating equation

$$\begin{aligned} \sum_{i=1}^{N}\sum_{k=1}^{K}[d_{k-1}(\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-1)}) - E(d_{k-1}(\bar{L}^*_{ik-1}, \bar{A}^*_{ik-1})|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)})]\circ \\ V^{-1} \circ [U^*_{i(k-1)}(\tau) - E(U^*_{i(k-1)}(\tau)|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)})] = 0, \end{aligned} \tag{29}$$

where $d_{k-1}(\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-1)})$ is an arbitrary $p \times (K-k+1)$-dimensional function, with p the dimension of τ, and $V^{-1} = Var(U^*_{i(k-1)}(\tau) - E(U^*_{i(k-1)}(\tau)|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)}))^{-1}$.

This estimating equation essentially sets the sum across time points of the conditional covariances between $U^*_{i(k-1)}(\tau)$ and the given function $d_{k-1}(\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-1)})$, given $\bar{L}^*_{i(k-1)}$, $\bar{A}^*_{i(k-2)}$ to zero (Vansteelandt & Joffe, 2014). When there is homoscedasticity in V, then local semiparametric efficiency under the SNMM is attained upon choosing

$$d_{k-1}(\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-1)}) = E\left[\frac{\partial U^*_{i(k-1)}(\tau^*)}{\partial \tau} | \bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-1)}\right]. \tag{30}$$

Solving the estimating equation (29) requires a parametric model $\mathcal{A}$ for the treatment/predictor A^*_{ik}: $f(A^*_{i(k-1)}|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)}; \eta)$, at $k = 1, \ldots, K$. It also requires a parametric model $\mathcal{B}$ for the conditional mean of $U^*_{i(k-1)}(\tau)$, namely, $f(U^*_{i(k-1)}(\tau)|\bar{L}^*_{i(k-1)}, \bar{A}^*_{i(k-2)}; \kappa)$. When parameters η and κ are variation-independent, G-estimators that solve Equation (29), obtained by substituting η and κ with consistent estimators, are doubly robust (Robins & Rotnitzky, 2001), meaning they are consistent when either model $\mathcal{A}$ or model $\mathcal{B}$ is correctly specified. Some alternatives that demand correct specification of model $\mathcal{B}$ have been proposed (see Vansteelandt & Joffe, 2014).

5 SIMULATION STUDIES

5.1 Method

This section describes a Monte Carlo simulation for systematically investigating how effectively the proposed method using calculated within-person variability scores can recover causal parameters, and presents comparisons of estimation performance versus other potential (centering) methods. For simplicity, we consider the case where causal effects are homogeneous among persons and interactions or moderation effects with observed confounders are not present. Data are generated by a linear model with normal residuals, as in the RI-CLPM.

First, initial within-person variability scores (Y^*_{i0}, A^*_{i0} and L^*_{i0}) are generated from a multivariate normal with variance 10 and covariance 3. Then, within-person variability scores at succeeding times are sequentially generated via a first-order linear autoregressive model with the stationarity assumption

$$\begin{aligned} Y^*_{ik} &= 0.40Y^*_{i(k-1)} + 0.40A^*_{i(k-1)} + 0.10L^*_{i(k-1)} + d^{(Y)}_{ik}, \\ A^*_{ik} &= 0.20Y^*_{ik} + 0.40A^*_{i(k-1)} + 0.30L^*_{ik} + d^{(A)}_{ik}, \\ L^*_{ik} &= 0.20Y^*_{i(k-1)} + 0.20A^*_{i(k-1)} + 0.50L^*_{i(k-1)} + d^{(L)}_{ik}, \end{aligned} \tag{31}$$

indicating that the average first-order causal effect from $A^*_{i(k-1)}$ is 0.40. The second-order causal effect from $A^*_{i(k-2)}$ can be calculated using path tracing rules as $0.40 \times 0.40 + 0.10 \times 0.20 = 0.18$. Therefore, in linear SNMMs, $E(Y^{*\bar{a}^*_{i(k-3)},a^*_{i(k-2)},a^*_{i(k-1)}}_{ik} - Y^{*\bar{a}^*_{i(k-3)},0,0}_{ik} | \bar{L}^*_{i(k-1)} = \bar{l}^*_{i(k-1)}, \bar{A}^*_{i(k-1)} = \bar{a}^*_{i(k-1)}) = 0.40a^*_{i(k-1)} + 0.18a^*_{i(k-2)}$, and in MSMs $E(Y^{*a^*_{i(k-2)},a^*_{i(k-1)}}_{ik}) = E_{\bar{a}^*_{i(k-3)}}(Y^{*\bar{a}^*_{i(k-3)},a^*_{i(k-2)},a^*_{i(k-1)}}_{ik}) = 0.40a^*_{i(k-1)} + 0.18a^*_{i(k-2)}$. Because no moderation effects are assumed, parameter values are equivalent between MSMs and SNMMs. We assume a goal of estimating the joint effects of past treatments/predictors on the outcome at $k = 1, \ldots, 4$. Let $b_{kk'}$ ($k \geq k'$) be a k'-th order causal effect for the outcome at t_k in the population. Namely, $b_{11} = 0.40$, $b_{21} = 0.40$, and $b_{22} = 0.18$. Third- and fourth-order causal effects can be calculated in a similar manner as 0.09 and 0.0486, respectively. Thus, $b_{31} = 0.40$, $b_{32} = 0.18$, $b_{33} = 0.09$, $b_{41} = 0.40$, $b_{42} = 0.18$, $b_{43} = 0.09$, and $b_{44} = 0.0486$, resulting in a total of 10 different causal parameters. Variance of normal residual d was set to 5 for each variable, making the variance of within-person variability scores for each variable become almost 10 at each time point (the proportion of variance explained in Equation (31) becomes almost 50%).

Three kinds of stable trait factors ($I^{(Y)}_i$, $I^{(A)}_i$, and $I^{(L)}_i$) are generated by multivariate normals with a correlation of 0.3. Observed values are then generated using the relation of

Equation (1),

$$Y_{ik} = I_i^{(Y)} + Y_{ik}^*, \quad A_{ik} = I_i^{(A)} + A_{ik}^*, \quad L_{ik} = I_i^{(L)} + L_{ik}^*, \tag{32}$$

where temporal group means are set to zero ($\mu_k^{(Y)} = \mu_k^{(A)} = \mu_k^{(L)} = 0$).

In this simulation, we systematically changed the total number of persons $N = 200, 600, 1000$, the number of time points $K = 4, 8$, and the size of stable trait factor variances $\phi^2_{(Y)} = \phi^2_{(A)} = \phi^2_{(L)} = 10/9, 30/7, 10$. This setting of stable trait factor variances indicates that the proportion of this variance to that of observations becomes around 10%, 30%, and 50% at each time point.

By crossing these factors, we generated 200 simulation data for each combination of factors. For comparison, each simulation dataset was analyzed by MSMs and SNMs using four different scores: 1) True within-person variability scores (true factor score centering: e.g., $Y_{ik}^* = Y_{ik} - I_i^{(Y)}$ for Y), 2) observed scores (no centering, e.g., $\hat{Y}_{ik}^* = Y_{ik}$), 3) scores based on observed person-specific means (observed-mean centering, e.g., $\hat{Y}_{ik}^* = Y_{ik} - \bar{Y}_i$), and 4) within-person variability scores predicted by the proposed method (Equation 16). The no-centering method ignores the presence of stable traits as a potential time-invariant unobserved confounder. As previously noted, multilevel models often include observed mean-centered predictors A and confounders L. However, because these observed means include the components of both stable traits (between-person differences) and within-person variability, observed-mean centering fails to perfectly disentangle stable individual differences from within-person variability.

In the first step of the proposed method, a linear AR(1) structure that assumes time-varying autoregressive parameters and residual variances is specified for each variable. Although a true model (AR(K) structure) cannot be specified because of the identification problem, we confirmed that the AR(1) structure generally provides acceptable model fits under this parameter setting.

The results are discarded when improper solutions appear in the first step due to out-of-range parameter estimates (e.g., negative variances parameters). In the current simulation, less than 0.1% of all estimates produced such improper solutions. We also confirmed that improper solutions were not found in the second step of applying MSMs and SMMs. When applying MSMs, a first-order linear regression model is specified for treatment assignment model, namely, $f(A_t|A_{t-1}, L_t)$ (the correct specification). For SNMMs, models of both $\mathcal{A}$ and $\mathcal{B}$ are correctly specified.

Under each simulation condition, we calculated bias and root mean squared error (RMSE) of 10 kinds of causal effects estimates from MSMs and SNMMs: $\hat{b} = (\hat{b}_{11}, \hat{b}_{21}, \hat{b}_{22}, \hat{b}_{31}, \hat{b}_{32}, \hat{b}_{33}, \hat{b}_{41}, \hat{b}_{42}, \hat{b}_{43}, \hat{b}_{44})$. Because of the stationarity assumption in Equation (31), this investigation is essentially equivalent to the one for $\hat{b}^* = (\hat{b}_{51}, \hat{b}_{61}, \hat{b}_{62}, \hat{b}_{71}, \hat{b}_{72}, \hat{b}_{73}, \hat{b}_{81}, \hat{b}_{82}, \hat{b}_{83}, \hat{b}_{84})$ in the $K = 8$ condition.

The simulation was conducted in R, using the *lavaan* package (Rosseel, 2012) to estimate parameters by MLE in the first step and the *ipw* package for MSMs in the second step. In SNMMs, we solve Equation (29) via the Newton–Raphson method. Simulation code is available in the Online Supplemental Materials.

5.2 Results

Because of space limitations, Figure 1 shows only biases of causal effects estimates in MSMs and SNMMs when $\phi^2 = 10/9, 10$. Because differences in the N value were minor in terms of bias, here we only show the result when $N = 1000$. Results under other conditions are provided in the Online Supplemental Materials (Figures S2 and S3).

Figure 1 shows that true score conditions produce almost no biases in both MSMs and SNMMs. SNMMs show smaller RMSEs than MSMs on average (Figure S1). In the proposed method, estimates show biases because of the biased estimates of stable trait

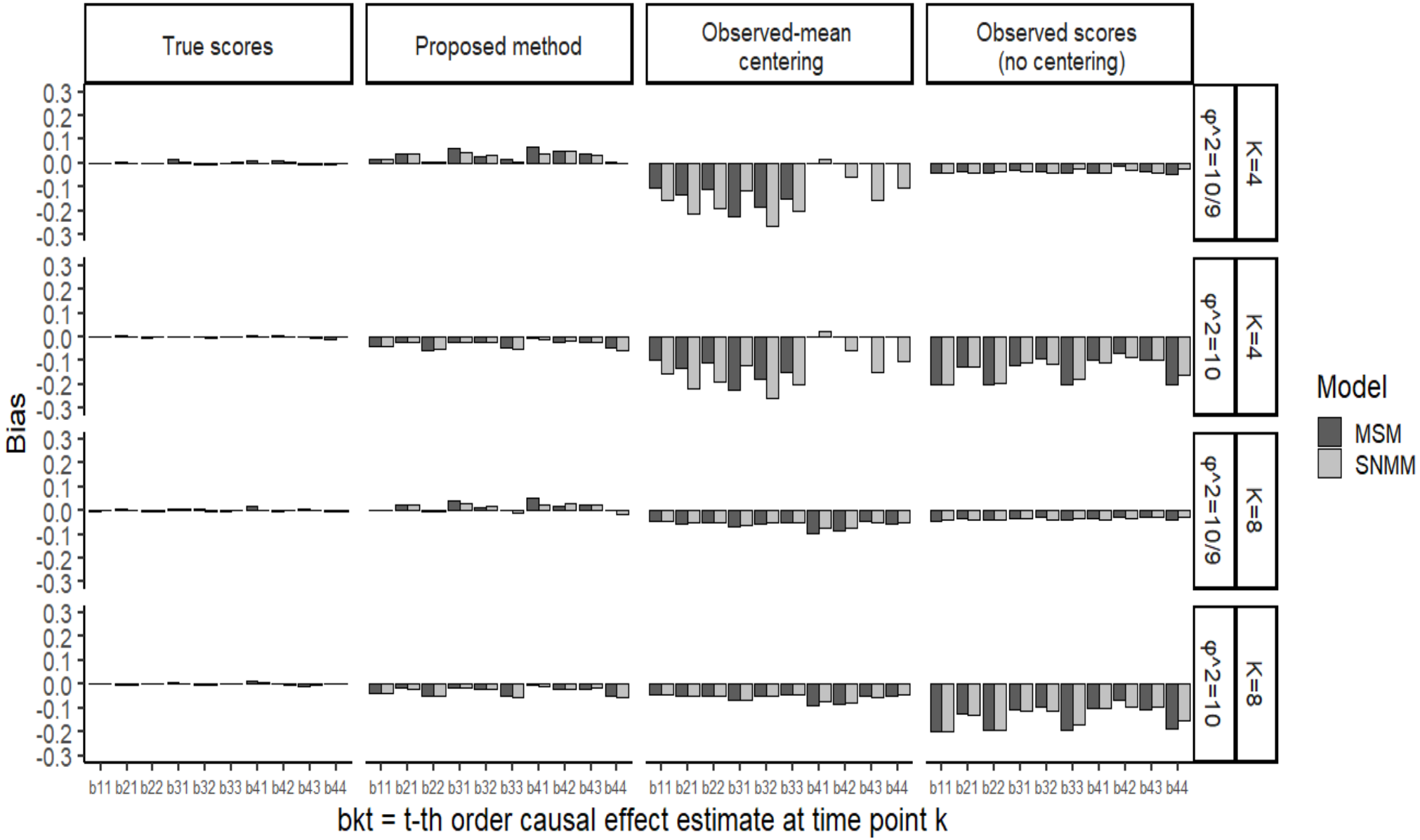

Figure 1. Biases in causal effect estimates ($N = 1{,}000$)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

factor (co)variances triggered by a model misspecification in the first step. However, the magnitude of biases is much smaller than in the observed-mean centering and no-centering methods. SNMMs again show smaller RMSEs than do MSMs (Figure S1). The observed-mean centering method shows negative biases, and its magnitude becomes larger when $K = 4$. This result is caused by negatively biased covariances in variables resulting from subtracting observed means from observations, and this effect increases as K decreases. Another critical aspect of this method is that linear dependence prevents identification of joint effects of all past treatments on Y_K (in this case, b_{41}, b_{42}, b_{43}, b_{44}). We therefore do not recommend use of observed-mean centering. The no-centering method shows serious negative biases when ϕ^2 is not small, indicating that ignoring the presence of stable traits is critical to estimating causal effects. Magnitudes of stable trait factor variances can vary depending on the measured outcomes and study period, but many studies applying the RI-CLPM have shown significant and moderate to large sizes of $\hat{\phi}^2$ (e.g., the proportion of stable trait factor variance to that of observations is above 30%). The following application also demonstrates large stable trait factor variances estimates.

In supplemental analyses, we additionally explored the performance of the methods under different parameter settings, as well as different model specifications in the first step. From this, we find similar tendencies in the results (Figures S4–S7): (1) SNMMs show smaller RMSEs than do MSMs, and (2) the proposed method shows adequate performance in terms of biases and RMSEs, and it works better than the no-centering method (especially when ϕ is larger) and the observed-mean centering method (especially when K is smaller). We also investigated the performance of linear correlation preserving predictor ($\hat{I}_i^{(Y)}$ in Equation 21) centering (e.g., $\hat{Y}_{ik}^{*} = Y_{ik} - \hat{I}_i^{(Y)}$), confirming that the proposed method could work much better than this method on average (Figures S4–S7).

6 EMPIRICAL APPLICATION

This section describes an empirical application of the proposed method using data from the Tokyo Teen Cohort (TTC) study (Ando et al., 2019). TTC was a multidisciplinary longitudinal cohort study on the psychological and physical development of adolescents who were 10 years old at enrollment and lived in municipalities in the Tokyo metropolitan area (Setagaya, Mitaka, Chofu). Datasets were collected in three waves: from 2012 to 2015, from 2014 to 2017, and from 2017 to 2019 (i.e., $K = 2$). In total, 3,171 children participated in the survey. See Ando et al. (2019) for more detailed information about measured variables, participant recruitment, and demographic characteristics of participants in the TTC study.

In this example, we estimate the (joint) causal effects of time-varying sleep duration (A) on later depressive symptoms (Y) in adolescents. Several epidemiological studies have suggested a relationship between sleep habits (sleep duration, bedtime, and bedtime regularity) and mental health status (depression and anxiety) in adolescents. For example, Matamura et al. (2014) applied the CLPM to data from 314 monozygotic twins living in Japan, and showed that sleep duration had significant associations with mental health indices, controlling for genetic and shared environmental factors. However, to the authors knowledge, no studies have investigated this relation under within-person variability score-based inference that accounts for stable traits in sleep duration and symptoms.

The Short Mood and Feelings Questionnaire (SMFQ; Angold et al., 1995) was used to measure depression in adolescents (Y). The SMFQ consists of 13 items assessing depressive symptoms rated on a 3-point scale (0: *not true*, 1: *sometimes true*, 2: *true*) regarding feelings and actions over the preceding 2 weeks. Higher SMFQ scores suggest more severe symptoms. These data were measured at home by self-report questionnaires. In this example, sleep duration in hours (A) was measured by the question "How long do you

usually sleep on weekdays?" Confounders were body mass index (BMI; L_B) and bedtime (L_A), which was measured by the question "When do you usually go to bed on weekdays?" Because many adolescents reported no problems for all items on the SMFQ, the score distribution was positively skewed. In the present example, we focus on the clinical group comprising $N = 416$ adolescents (13.1%) with SMFQ scores of 6 or higher during the study. Katon, Russo, Richardson, McCauley, and Lozano (2008) reported 80% sensitivity and 81% specificity at this cut-off for diagnosis of major depression based on the Computerized Diagnostic Interview Schedule for Children (C-DISC). Missing data were primarily due to dropout. Of the 416 samples, 113 adolescents provided all three responses in the study. Descriptive statistics of sleep duration, SMFQ score, bedtime, and BMI are available in the Online Supplemental Materials (Table S1).

In the first step, we use generalized least squares in the *lavaan* package to estimate the model parameters for each variable. To model within-person variability scores in each variable, we use an AR(1) structure that assumes time-varying autoregressive parameters and residual variances. Let P_i be the total number of variables observed in adolescent i. $P_i \times P_i$ weights W_i are calculated from estimated parameters under the assumption of MAR. Within-person variability scores X_i^* are then calculated using this weight and observations $X_{i,obs}$ for adolescent i as $X_i^* = W_i^t X_{i,obs}$.

Causal effects of sleep duration at 10 and 12 years old (A_0 and A_1) on later depressive symptoms (SMFQ scores Y_1 and Y_2) are estimated using calculated within-person variability scores by linear SNMM. In linear SNMM, blip functions and $U^*(\tau)$ are set as in Equations (25) and (27), except that the two confounders L_A and L_B are present in this example. When applying SNMMs, models of both $\mathcal{A}$ and $\mathcal{B}$ are specified using first-order linear regression models, as in the simulation. All calculated within-person variability scores were used in the analysis under the assumption of MAR.

We confirmed that the first step did not find improper solutions, and that current AR(1) models that assume time-varying parameters fit better than those that do not. Table S2 summarizes model fit indices and estimated parameters in this step. All stable trait factor variance estimates are significant, indicating the necessity of controlling for stable traits. Specifically, proportions of estimated stable trait factor variances to total variances (at $k = 0$) are 24.5%, 54.5%, 48.2%, and 74.8% for Y, A, L_A, and L_B, respectively.

Table 1 provides the estimation results of causal effects, along with estimates from the no-centering and observed-mean centering methods for comparison. As seen in Table 1, the proposed method reveals that longer sleep duration at 12 years old (A_1) has a positive effect ($\hat{\tau} = -2.704$, 95%CI [-4.938,-0.470], $p < .05$) on later depressive symptom at 14 years old (Y_2), but this estimate is not significant in the no-centering and observed mean score-centering methods. Similar positive effects of sleep duration were found in previous studies (Matamura et al., 2014), but the present analysis newly shows this causal effect at the within-person level by controlling for stable traits of persons as a potential time-invariant unobserved confounder. When the no-centering method is applied, the causal effect estimate of A_0 on Y_1 is significant, showing that longer sleep duration at 10 years old has a negative effect ($\hat{\tau} = 1.693$, 95% CI [0.405,2.981], $p < .05$) on later depressive symptoms at 12 years old. However, because magnitudes of the estimated stable trait factor variances were moderate or large for all variables, causal effect estimates in the no-centering method are unreliable and might be seriously biased.

In supplemental analyses, we confirmed that major findings did not change even when using complete data (N=114) and a different cutoff for SMFQ (Angold et al., 1995). Statistical significance as well as sign and magnitude in causal effect estimates might change according to choice of calculation methods for within-person variability scores, and ignoring the presence of stable trait variances might lead to wrong conclusions (Tables S2–S5).

Table 1: Causal effects estimates (SEs) of sleep duration on depression (SMFQ) (N=416).

	Proposed method	Observed-mean centering	Observed scores (no centering)
$Sleep_1 \rightarrow SMFQ_2$	**-2.704 (1.140)**	0.095 (0.869)	-1.492 (1.080)
$(Sleep_1 \times Bedtime_1) \rightarrow SMFQ_2$	-0.603 (1.416)	0.916 (1.857)	0.336 (1.057)
$(Sleep_1 \times BMI_1) \rightarrow SMFQ_2$	-0.442 (0.638)	-0.748 (1.169)	-0.532 (0.399)
$Sleep_0 \rightarrow SMFQ_2$	-0.293 (1.179)	-0.309 (1.021)	0.185 (1.117)
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_2$	2.278 (1.918)	-3.012 (1.784)	1.169 (1.792)
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_2$	**-1.856 (0.780)**	-0.251 (0.986)	0.306 (0.287)
$Sleep_0 \rightarrow SMFQ_1$	0.702 (0.686)	0.279 (0.572)	**1.693 (0.657)**
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_1$	0.315 (1.177)	1.037 (1.069)	-0.918 (0.920)
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_1$	0.021 (0.473)	0.773 (0.572)	-0.101 (0.212)

Note: Bold font indicates statistical significance.

7 CONCLUDING REMARKS

We proposed a two-step semiparametric estimation method for within-person variability score-based causal inference to estimate joint effects of time-varying continuous treatments. In the first step, a within-person variability score for each person, which is disaggregated from the stable trait factor score, is calculated using weights based on the best linear correlation preserving predictor through SEM. Causal parameters are then estimated by MSMs or SNMs, using calculated within-person variability scores. By taking this within-person variability score-based causal inference approach, in which stable trait factors as potential time-invariant unobserved confounders are controlled for, causal effects of treatments/predictors can be estimated with less risk of violating the identifiability assumption of sequential ignorability. The proposed method can be considered as semiparametric approach that synthesizes the two traditions of factor analysis/SEM in psychometrics and the potential outcome approach (MSMs/SNMs) in epidemiology. Our approach is more flexible than the RI-CLPM in modeling how observed confounders are connected to outcomes and treatments/predictors. We particularly emphasize the utility of SNMs with G-estimation, because of its doubly robust property to model errors, along with flexibility in that it

allows investigation of moderation effects of treatments with observed confounders. One caveat is that the proposed approach (as well as the RI-CLPM) demands longitudinal data with three or more time points ($K \geq 2$) for model identification in measurement models, specified in the first step.

Through simulation and empirical application, we illustrated that ignoring the presence of stable traits might lead to wrong conclusions in causal effects. We also confirmed that the proposed approach is superior to observed- mean centering, which is often used in applications of multilevel models. Especially when K is small, observed-mean centering showed serious negative biases in causal effect estimates. Considering that most research applying the RI-CLPM to uncover within-person relations use longitudinal data with two or three time points ($K = 1, 2$; e.g., Usami, Todo, & Murayama, 2019), observed-mean centering cannot be recommended.

We assumed continuous variables in this paper, but we can extend this methodology to categorical variables. For binary and ordered categorical variables, one simple method is to use estimated tetrachoric or polychoric correlations as input for factor analysis and then to calculate within-person variability scores using estimated stable trait factor and unique factor (within-person variability scores) variances. There are alternative methods for conducting factor analysis for ordinal variables (e.g., J*ö*reskog & Moustaki, 2001), but whatever method is used, rescaling stable trait factor and unique factor score variances might be required to make calculated within-person variability scores interpretable. Using variances of original (categorical) data should be one option for this purpose. However, estimation performance for causal parameters and the influence of biased tetrachoric or polychoric correlations caused by misspecified distributions need to be investigated in future research.

Because we take an SEM approach in the first step, accounting for measurement errors in

observations, which is closely related to violation of the consistency assumption, is feasible under the parametric assumption. Although we expect longitudinal data with large K are required for precisely estimating measurement errors variances, we plan to investigate how the proposed method works under measurement models that include measurement errors.

This paper opens a new avenue for exploration of various within-person variability-based research questions. For example, use of within-person variability scores can be extended to cases where one is interested in uncovering reciprocal effects (e.g., Usami, Murayama, & Hamaker, 2019) and mediation effects (e.g., Goldsmith et al., 2018; Tchetgen Tchetgen & Shpitser, 2012), as well as to hierarchical continuous time modeling (Driver & Voelkle, 2018). We hope the proposed method helps in exploring various questions of causality in longitudinal design and guiding better decision-making for researchers.

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Online Supplemental Material

Table S1: Means (SDs) of variables in the dataset.

	SMFQ	Sleep duration (hrs)	Bedtime (pm)	BMI
$k = 0$ (year10)	10.72 (4.13)	8.96 (0.65)	9:51 (40 min)	17.20 (2.83)
$k = 1$ (year12)	11.26 (4.67)	8.41 (0.80)	10:25 (49 min)	18.63 (3.15)
$k = 2$ (year14)	11.72 (4.99)	7.62 (0.95)	11:19 (57 min)	20.50 (3.32)

Table S2: Parameter estimates (SEs) and model fit indices at step 1 ($K = 2$).

	SMFQ (Y)	Sleep duration (A)	Bed time (L_A)	BMI (L_B)
Number of parameters	6	6	6	6
Degree of freedom	0	0	0	0
CFI	1	1	1	1
RMSEA	0	0	0	0
SRMR	0	0	0	0
Regression coefficient $Y_0^* \to Y_1^*$	-0.112 (0.161)	0.143 (0.254)	0.581 (0.180)	0.543 (0.264)
Regression coefficient $Y_1^* \to Y_2^*$	-0.046 (0.146)	0.208 (0.136)	0.357 (0.124)	0.484 (0.228)
$V(Y_0^*)$	15.928 (2.812)	0.224 (0.061)	0.268 (0.082)	2.919 (1.132)
Residual variances of Y_1^*	17.733 (3.716)	0.385 (0.072)	0.394 (0.048)	1.967 (0.565)
Residual variances of Y_2^*	21.448 (3.419)	0.632 (0.075)	0.601 (0.070)	1.486 (0.389)
$V(I)$	5.158 (1.977)	0.268 (0.064)	0.249 (0.083)	8.683 (1.692)

Note: Model fits are perfect because saturated models are used.

Table S3: Causal effect estimates (SEs) of sleep duration on depression (SMFQ) (N=113). (SMFQ cutoff = 6; Complete data by listwise deletion)

	Proposed method	Observed-mean centering	Observed scores (no centering)
$Sleep_1 \rightarrow SMFQ_2$	**-6.013 (1.523)**	0.175 (0.881)	-1.695 (1.110)
$(Sleep_1 \times Bedtime_1) \rightarrow SMFQ_2$	0.914 (2.744)	0.983 (1.910)	0.432 (1.127)
$(Sleep_1 \times BMI_1) \rightarrow SMFQ_2$	**-1.482 (0.544)**	-0.641 (1.187)	-0.568 (0.414)
$Sleep_0 \rightarrow SMFQ_2$	**-5.016 (1.950)**	-0.544 (1.049)	0.412 (1.166)
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_2$	**19.127 (5.241)**	-2.255 (1.791)	**6.612 (1.799)**
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_2$	-0.159 (0.575)	-0.571 (0.894)	0.110 (0.311)
$Sleep_0 \rightarrow SMFQ_1$	1.943 (1.589)	0.515 (1.026)	1.060 (1.109)
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_1$	-0.533 (4.272)	0.657 (1.751)	-0.889 (1.711)
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_1$	0.034 (0.469)	1.161 (0.874)	-0.040 (0.296)

Note: Bold font indicates statistical significance.

Table S4: Causal effects estimates (SEs) of sleep duration on depression (SMFQ) (N=254). (SMFQ cutoff = 8)

	Proposed method	Observed-mean centering	Observed scores (no centering)
$Sleep_1 \rightarrow SMFQ_2$	-	0.646 (1.000)	-1.803 (1.065)
$(Sleep_1 \times Bedtime_1) \rightarrow SMFQ_2$	-	1.390 (2.416)	-0.840 (0.910)
$(Sleep_1 \times BMI_1) \rightarrow SMFQ_2$	-	-0.745 (1.763)	-0.045 (0.430)
$Sleep_0 \rightarrow SMFQ_2$	-	-0.639 (1.199)	-0.884 (1.191)
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_2$	-	0.447 (2.525)	1.915 (1.775)
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_2$	-	-0.541 (1.060)	-0.012 (0.246)
$Sleep_0 \rightarrow SMFQ_1$	-	0.668 (0.627)	**1.802 (0.916)**
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_1$	-	0.169 (1.525)	-0.911 (1.317)
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_1$	-	0.770 (0.695)	-0.148 (0.240)

Note: Hyphens indicate estimates were not obtained because of nonconvergence.

Table S5: Causal effects estimates (SEs) of sleep duration on depression (SMFQ) (N=60). (SMFQ cutoff = 8; Complete data by listwise deletion)

	Proposed method	Observed-mean centering	Observed scores (no centering)
$Sleep_1 \rightarrow SMFQ_2$	**-4.012 (1.057)**	-0.418 (1.035)	-2.056 (1.120)
$(Sleep_1 \times Bedtime_1) \rightarrow SMFQ_2$	1.153 (1.573)	-0.964 (2.571)	-1.074 (1.016)
$(Sleep_1 \times BMI_1) \rightarrow SMFQ_2$	0.017 (0.579)	-1.415 (1.778)	0.110 (0.504)
$Sleep_0 \rightarrow SMFQ_2$	**-5.258 (1.561)**	-1.426 (1.127)	-0.088 (1.255)
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_2$	2.246 (3.087)	-1.139 (2.445)	**6.360 (2.112)**
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_2$	0.443 (0.475)	-0.255 (0.869)	-0.264 (0.319)
$Sleep_0 \rightarrow SMFQ_1$	2.179 (1.782)	1.125 (1.149)	1.476 (1.402)
$(Sleep_0 \times Bedtime_0) \rightarrow SMFQ_1$	0.035 (3.524)	2.065 (2.493)	0.532 (2.359)
$(Sleep_0 \times BMI_0) \rightarrow SMFQ_1$	-0.081 (0.543)	0.702 (0.886)	-0.165 (0.357)

Note: Bold font indicates statistical significance.

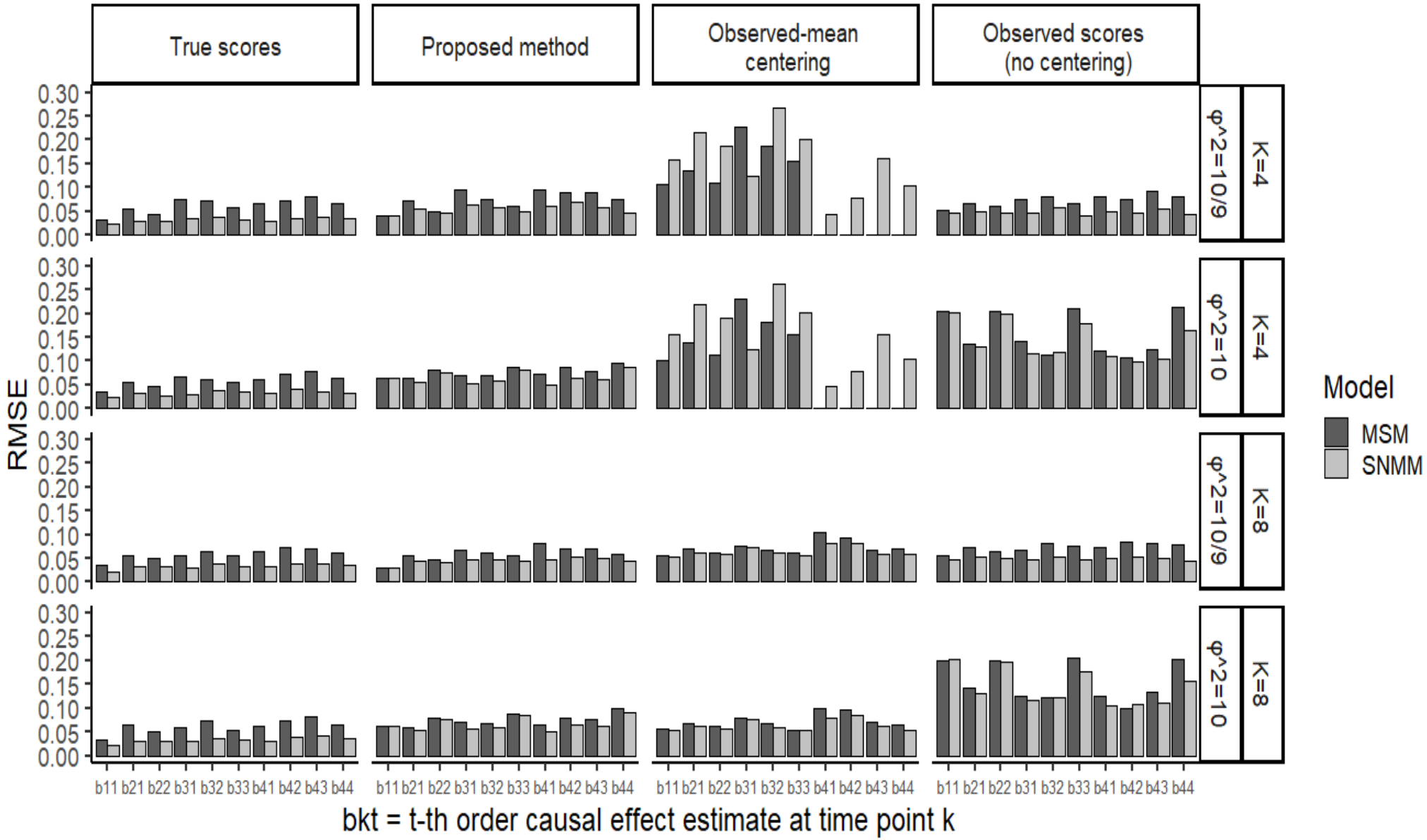

Figure S1. RMSE of causal effect estimates (N = 1,000)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

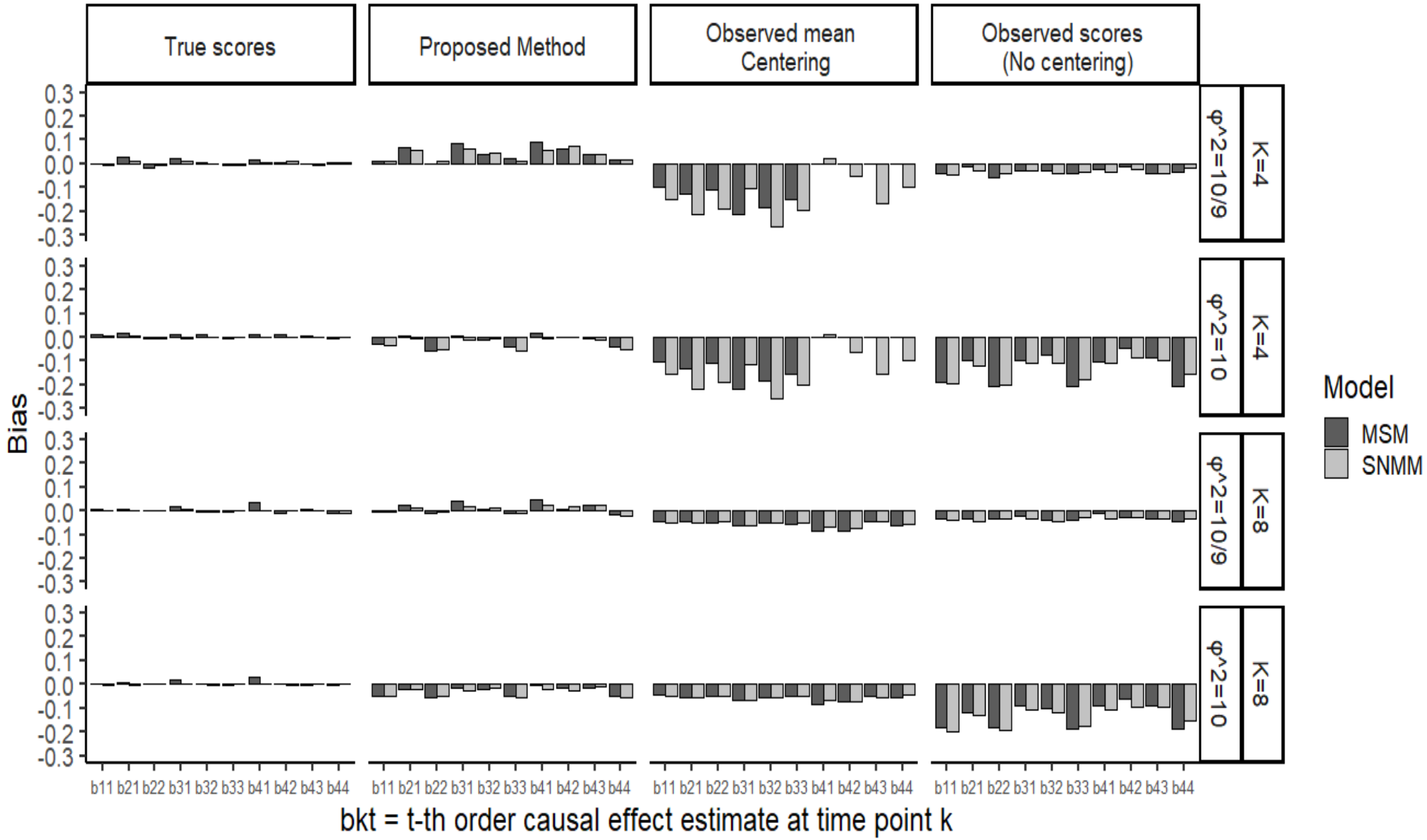

Figure S2. Biases of causal effect estimates ($N = 200$)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

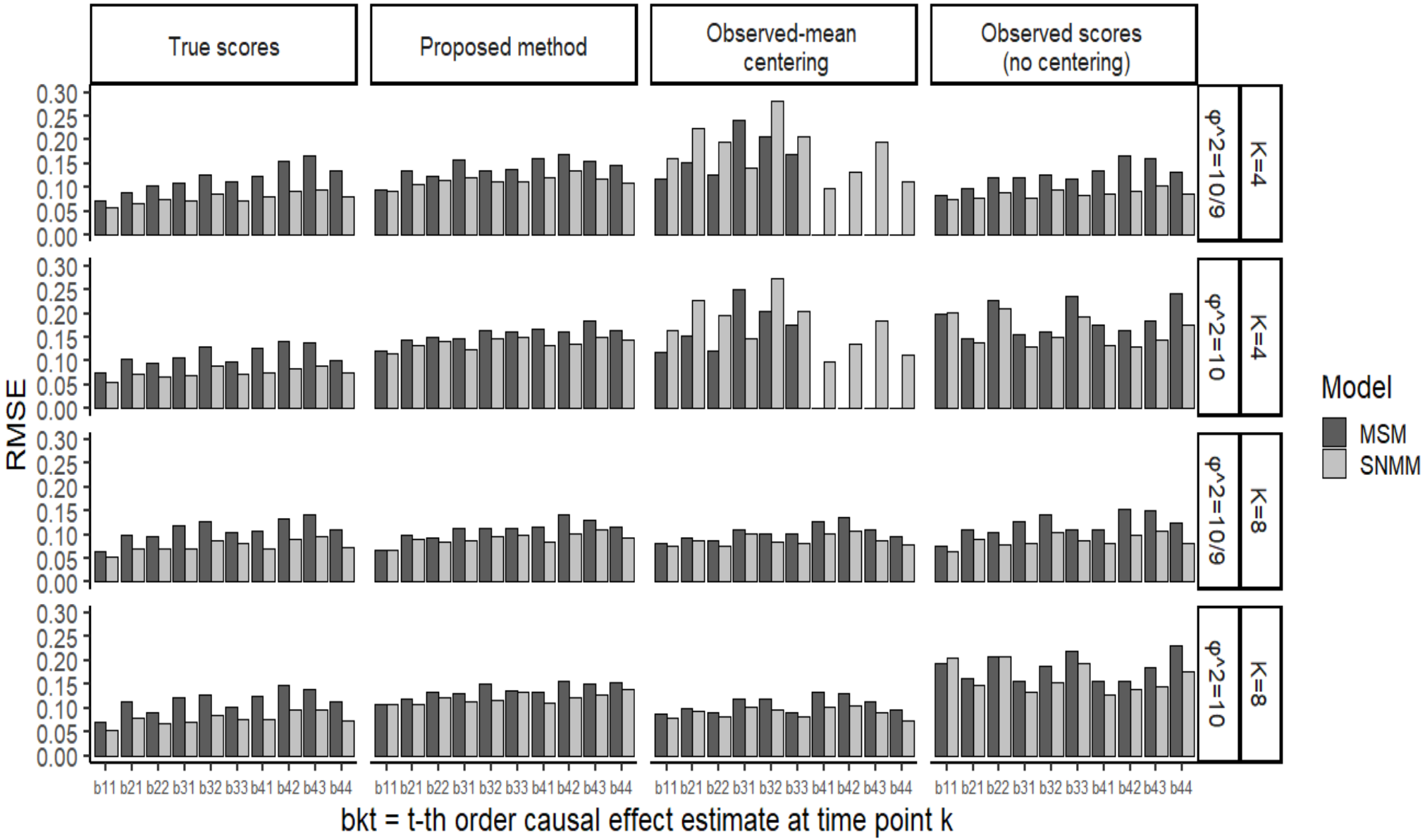

Figure S3. RMSE of causal effect estimates ($N = 200$)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

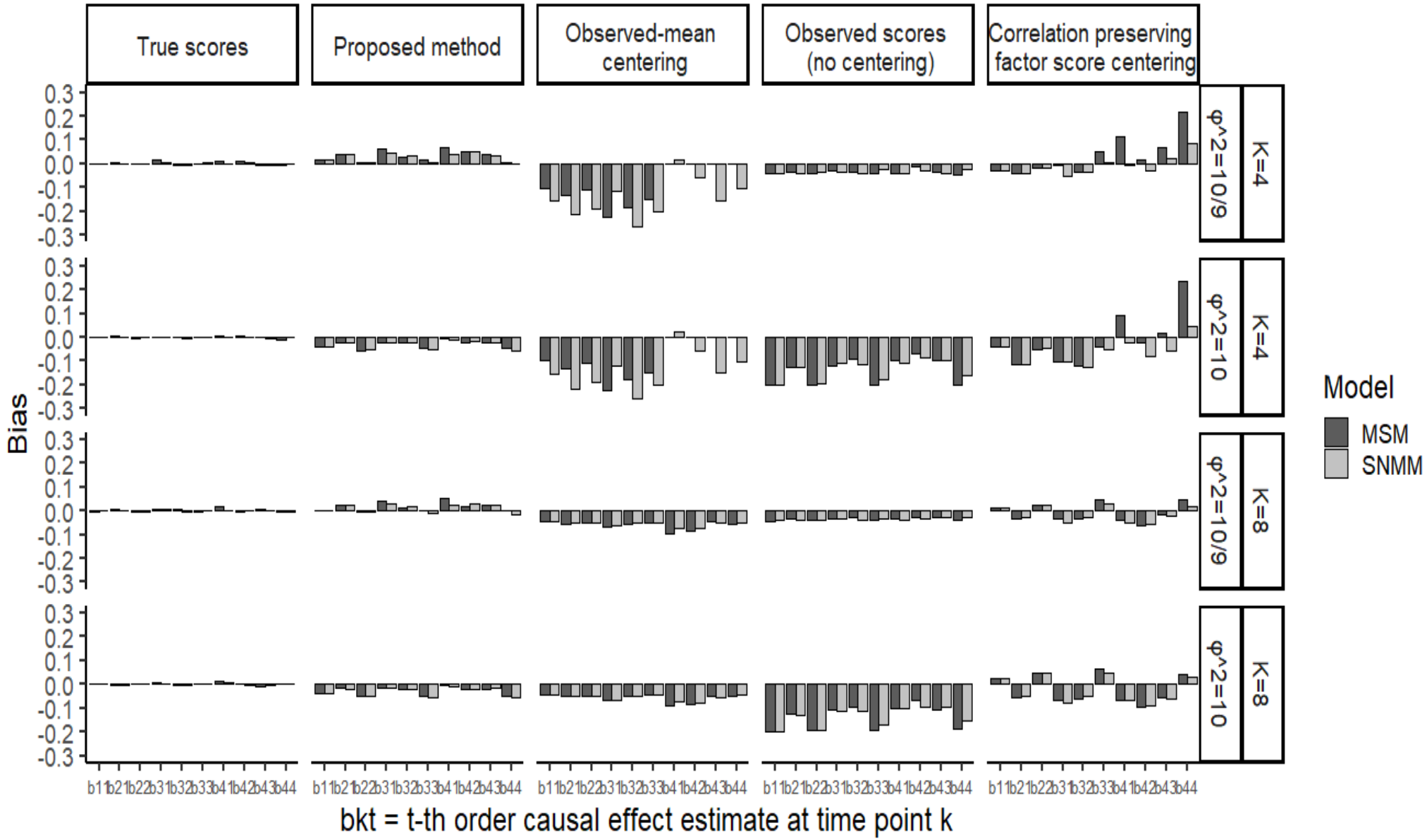

Figure S4. Biases of causal effect estimates that include the condition of correlation preserving factor score centering ($N = 1{,}000$)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

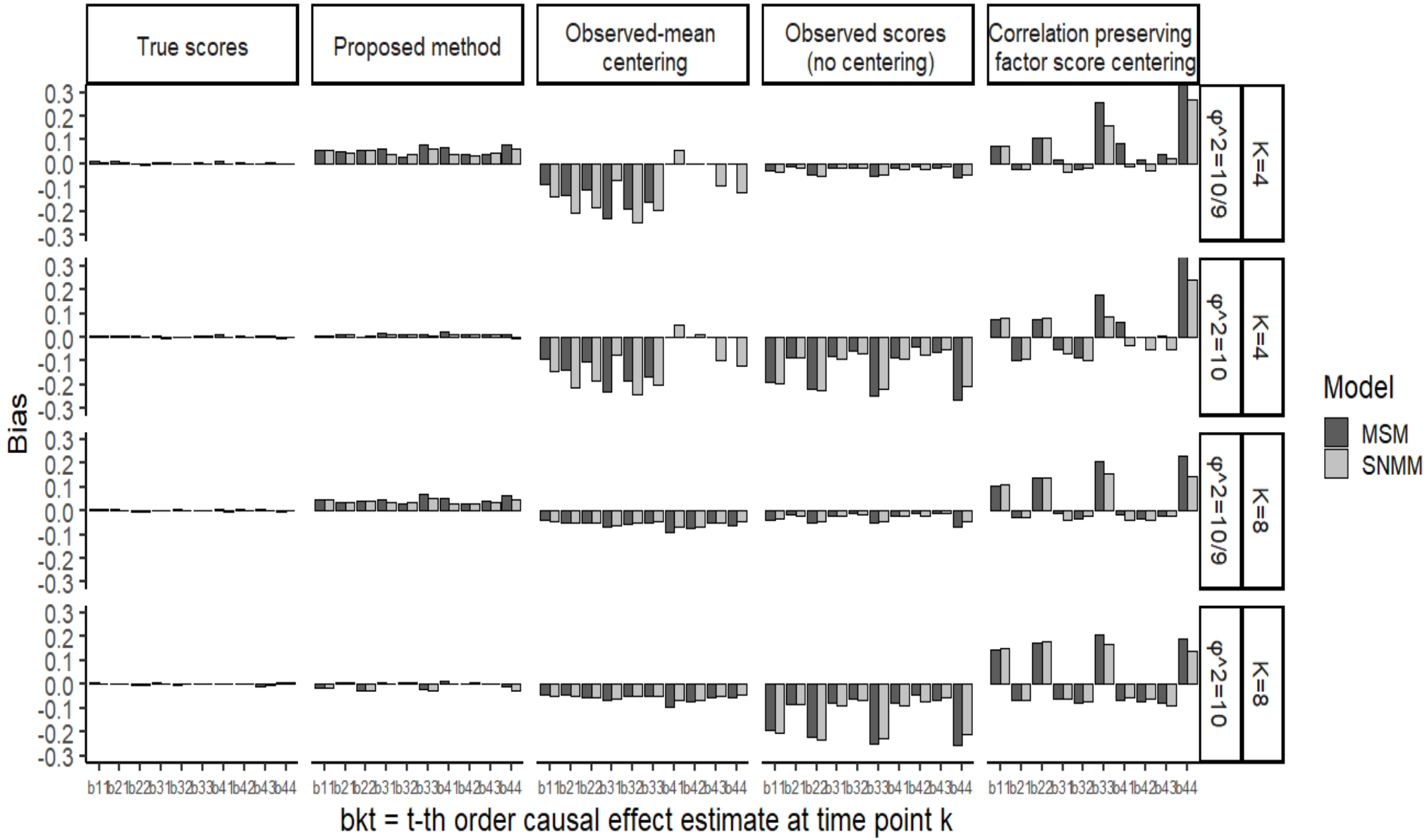

Figure S5. Biases of causal effect estimates (N = 1,000, variance of residuals = 10)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

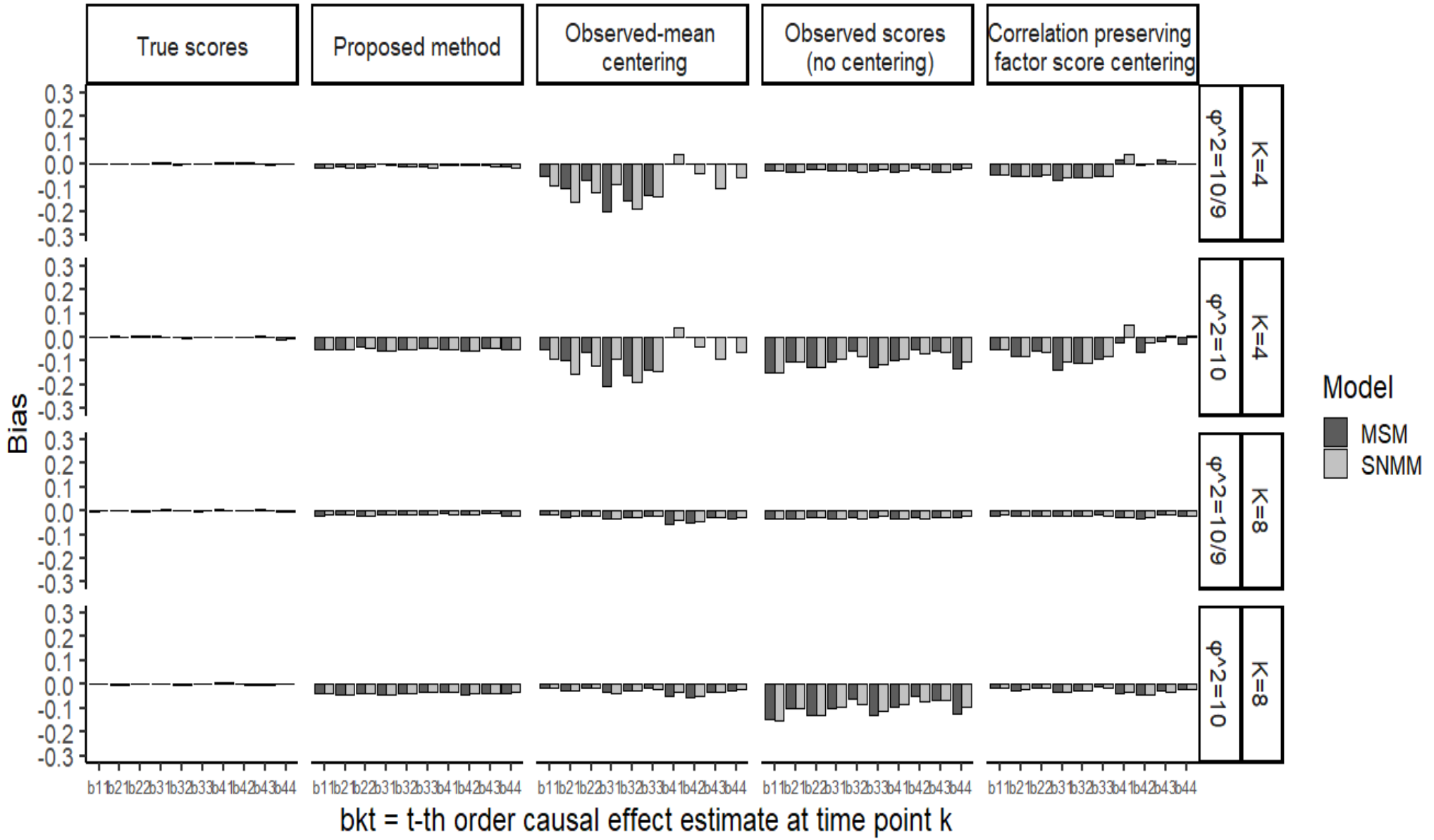

Figure S6. Biases of causal effects estimates ($N = 1{,}000$, variance of residuals = 5, Y_1^*=0.2Y_0^*+0.3A_0^*+0.2*L_0^*+$d(Y)$, A_1^*=0.2Y_0^*+0.3A_0^*+0.2L_0^* + $d(A)$, L_1^*=0.2Y_0^*+0.2A_0^*+0.4L_0^* + $d(L)$, proportions of variances in Equation (31) of the main manuscript are almost 30%)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

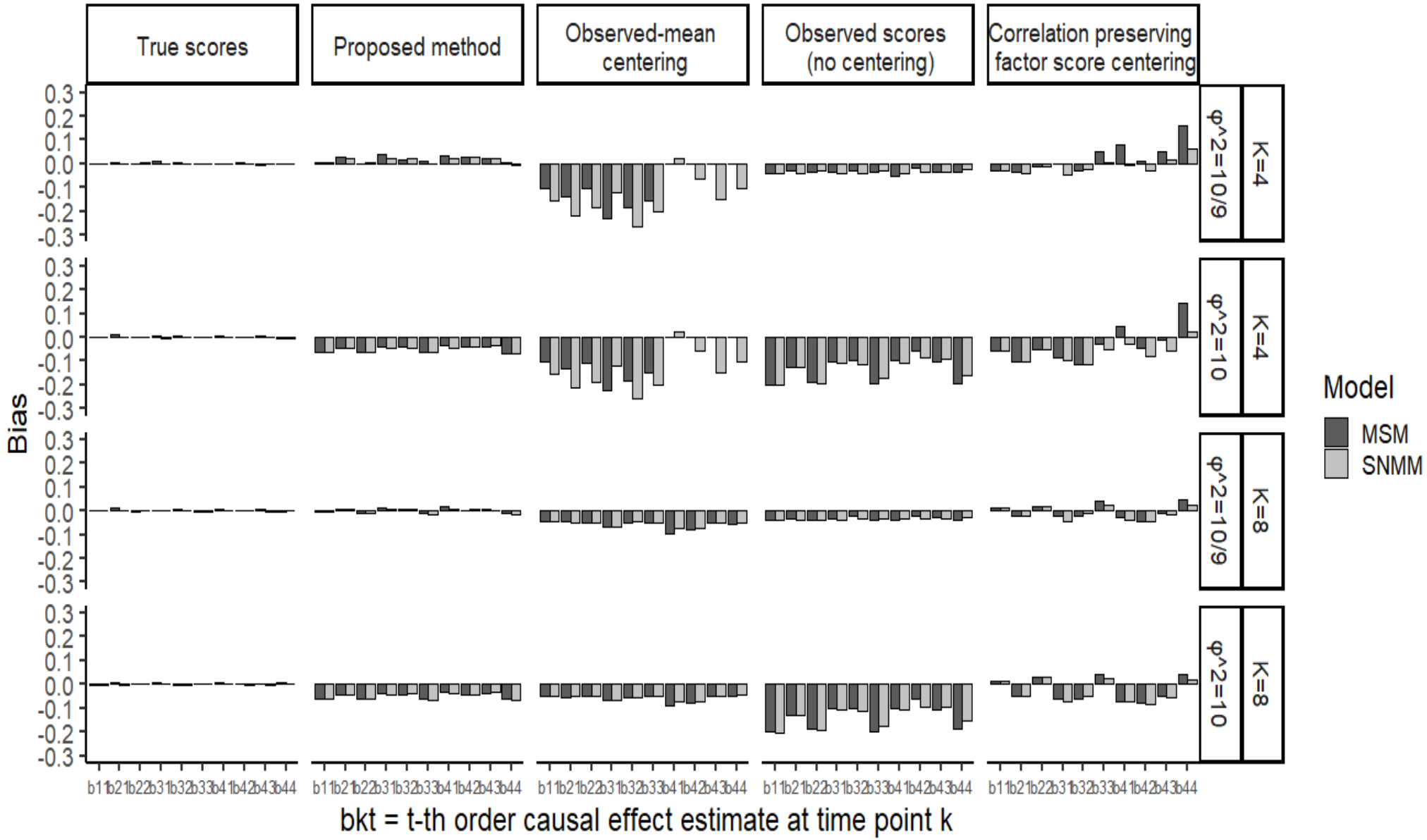

Figure S7. Biases of causal effects estimates (N = 1,000, AR(2) effects are included in step 1)

Note: Because of rank deficiency, estimates of $b_{41}, b_{42}, b_{43}, b_{44}$ for K=4 are not available in marginal structural models with observed-mean centering.

Derivation of the weight matrix W in Equations (16) and (18) of the main manuscript

Let $X_i = (Y_i^t, A_i^t, L_i^t)^t$ be a $(3K+1) \times 1$ vector of observations. X_i is modeled using a stable trait factor (between-person difference) and a unique factor (within-person variability score) as

$$X_i = \mu + \Lambda I_i + X_i^*, \tag{A1}$$

where μ is a $(3K+1) \times 1$ mean vector, Λ is a $(3K+1) \times 3$ binary matrix (factor loading matrix) that connects stable trait factor $I = (I_i^{(Y)}, I_i^{(A)}, I_i^{(L)})^t$ and an observation for each variable, and X_i^* is a $(3K+1) \times 1$ vector of within-person variability scores. Stable trait factors are uncorrelated with within-person variability scores ($Cov(I_i, X_i^*) = 0$). Unlike the standard factor analysis model, the covariance structure $\Psi = E(\hat{X}_i^* \hat{X}_i^{t*})$ is not diagonal.

We are concerned with linear prediction of a within-person variability score using $(3K+1) \times (3K+1)$ matrix W

$$\hat{X}_i^* = W^t(X_i - \mu), \tag{A2}$$

that preserves the covariance structure of within-person variability scores

$$E(\hat{X}_i^* \hat{X}_i^{*t}) = W^t E[(X_i - \mu)(X_i - \mu)^t]W = W^t \Sigma W = \Psi. \tag{A3}$$

Assume Ψ and Σ are positive definite and known because they are already estimated (as described in the section 4.1.1 of the main manuscript). Consider the following risk function defined as the trace of the usual mean squared error in within-person variability scores:

$$MSE(\hat{X}_i^*) = E\left[(\hat{X}_i^* - X_i^*)^t(\hat{X}_i^* - X_i^*)\right] = trE\left[(\hat{X}_i^* - X_i^*)(\hat{X}_i^* - X_i^*)^t\right] \tag{A4}$$

From relations (A1) and (A3), this risk function can be expressed as

$$MSE\left(\hat{X}_i^*\right) = trW^t\Sigma W + tr\Psi - 2tr\Psi W = 2tr\Psi - 2tr\Psi W. \quad \text{(A5)}$$

Therefore, minimization of (A5) is mathematically equivalent to that of $tr\Psi W$. From relation (A3), W can be expressed using a $(3K+1)\times(3K+1)$ orthogonal matrix Q as

$$W = \Sigma^{-1/2}Q\Psi^{1/2} \text{ for some } Q\text{: } (3K+1)\times(3K+1) \text{ such that } Q^tQ = I. \quad \text{(A6)}$$

From (A6), $tr\Psi W$ can now be expressed as

$$tr\Psi W = tr\Psi\Sigma^{-1/2}Q\Psi^{1/2} = tr\Psi^{3/2}\Sigma^{-1/2}Q. \quad \text{(A7)}$$

Consider the singular value decomposition of $\Sigma^{-1/2}\Psi^{3/2}$ as

$$\Sigma^{-1/2}\Psi^{3/2} = UDV^t, \quad \text{(A8)}$$

where D is a diagonal matrix, and matrices U and V satisfy $V^tV = VV^t = U^tU = UU^t = I$. From (A8), (A7) now becomes

$$tr\Psi^{3/2}\Sigma^{-1/2}Q = trVDU^tQ = trU^tQVD. \quad \text{(A9)}$$

From the relation $-1 \le [U^tQV]_{kk} \le 1$ $(k = 1,\cdots,3K+1)$, (A9) can be maximized (and (A5) can be minimized) when $Q = UV^t$. Therefore,

$$Q = UV^t = UDV^t(VD^2V^t)^{-1/2} = UDV^t(VDU^tUDV^t)^{-1/2} = \Sigma^{-1/2}\Psi^{3/2}(\Psi^{3/2}\Sigma^{-1}\Psi^{3/2})^{-1/2}. \quad \text{(A10)}$$

From (A6) and (A10), W can now be expressed as

$$W = \Sigma^{-1}\Psi^{3/2}(\Psi^{3/2}\Sigma^{-1}\Psi^{3/2})^{-1/2}\Psi^{1/2}. \quad \text{(A11)}$$

Therefore, the best linear predictor for within-person variability scores can be obtained as

$$\hat{X}_i^* = W^t(X_i - \mu) = \Psi^{1/2}(\Psi^{3/2}\Sigma^{-1}\Psi^{3/2})^{-1/2}\Psi^{3/2}\Sigma^{-1}(X_i - \mu), \quad \text{(A12)}$$

which corresponds to Equations (16) and (18) in the main manuscript.

R Code for Simulations (*K*=4 condition)

```
SIMULATION<-function(TTT,REPEAT){
#Store the simulation results in POINTEST and SEEST
POINTEST<-matrix(rep(0,3*4*2*3*106),3*4*2*3,106);SEEST<-matrix(rep(0,3*4*2*3*106),3*4*2*3,106)

#Download required packages
library(lavaan);library(MASS); library(geepack);library(survey);library(ipw);library(reshape);library(dplyr)

#For loops (aaaa associates sample size N, and dddd associates stable trait variances TRAITVAR)
for(aaaa in 1:3){; for(dddd in 1:3){;

#REPEAT=200 times in this simulation
COUNT<-0;while(COUNT<REPEAT){
N<-c(200,600,1000)[aaaa]; TRAITVAR<-c(10/9,30/7,10)[dddd]

#Generate simulation data
#Generate stable trait factor scores
MV<-mvrnorm(N,c(0,0,0), (TRAITVAR*0.3)+(TRAITVAR*0.7)*diag(3))
IY<- MV[,1]-mean(MV[,1]); IA<- MV[,2]-mean(MV[,2]);IL<-MV[,3] -mean(MV[,3])
#Generate within-person variability scores
dMV<-mvrnorm(N,c(0,0,0), 3+7*diag(3))
dY0<- dMV[,1]-mean(dMV[,1]); dL0<- dMV[,2]-mean(dMV[,2]) ; dA0<- dMV[,3]-mean(dMV[,3])
dL0A0<-dL0*dA0; dY0A0<-dY0*dA0
dY1<-0.4*dY0+0.1*dL0+0.4*dA0+0.00*dL0A0+0.0*dY0A0+rnorm(N,0,sqrt(5));dY1<-dY1-mean(dY1)

dL1<-0.2*dY0+0.5*dL0+0.2*dA0+rnorm(N,0,sqrt(5)) ; dL1<-dL1-mean(dL1)
dA1<-0.3*dL1+0.2*dY1+0.4*dA0+rnorm(N,0,sqrt(5));dA1<-dA1-mean(dA1);dL1A1<-dL1*dA1;dY1A1<-dY1*dA1
dY2<-0.4*dY1+0.1*dL1+0.4*dA1+0.00*dL1A1+0.0*dY1A1+rnorm(N,0,sqrt(5));dY2<-dY2-mean(dY2)

dL2<-0.2*dY1+0.5*dL1+0.2*dA1+rnorm(N,0,sqrt(5)) ; dL2<-dL2-mean(dL2)
dA2<-0.3*dL2+0.2*dY2+0.4*dA1+rnorm(N,0,sqrt(5));dA2<-dA2-mean(dA2);dL2A2<-dL2*dA2;dY2A2<-dY2*dA2
dY3<-0.4*dY2+0.1*dL2+0.4*dA2+0.00*dL2A2+0.0*dY2A2+rnorm(N,0,sqrt(5));dY3<-dY3-mean(dY3)

dL3<-0.2*dY2+0.5*dL2+0.2*dA2+rnorm(N,0,sqrt(5)) ; dL3<-dL3-mean(dL3)
dA3<-0.3*dL3+0.2*dY3+0.4*dA2+rnorm(N,0,sqrt(5));dA3<-dA3-mean(dA3);dL3A3<-dL3*dA3;dY3A3<-dY3*dA3
dY4<-0.4*dY3+0.1*dL3+0.4*dA3+0.00*dL3A3+0.0*dY3A3+rnorm(N,0,sqrt(5));dY4<-dY4-mean(dY4)

#Generate observed scores and dataset
Y0<-dY0+IY; Y1<-dY1+IY; Y2<-dY2+IY; Y3<-dY3+IY; Y4<-dY4+IY;
L0<-dL0+IL; L0<-dL0+IL; L1<-dL1+IL; L2<-dL2+IL; L3<-dL3+IL
A0<-dA0+IA; A0<-dA0+IA; A1<-dA1+IA; A2<-dA2+IA; A3<-dA3+IA
YY0<-Y0; YY1<-Y1;YY2<-Y2;YY3<-Y3;YY4<-Y4; LL0<-L0; LL1<-L1;LL2<-L2;LL3<-L3; AA0<-A0; AA1<-A1;AA2<-A2;AA3<-A3;
```

#STEP1 Predict within-person variability scores

#Step 1.1 Estimate model parameters for a variable Y

```
#lavaan code
FAIY <- 'TraitY =~1*Y0+1*Y1 +1*Y2+1*Y3 +1*Y4
dY0=~1*Y0;dY1=~1*Y1; dY2=~1*Y2; dY3=~1*Y3; dY4=~1*Y4
dY1~beta1*dY0; dY2~beta2*dY1; dY3~beta3*dY2; dY4~beta4*dY3;
TraitY~~0*dY0;TraitY~~0*dY1; TraitY~~0*dY2; TraitY~~0*dY3; TraitY~~0*dY4;
TraitY~~vy*TraitY;vy>0
dY1~~omega*dY1; dY2~~omega2*dY2;dY3~~omega3*dY3;dY4~~omega4*dY4
Y0~~ 0 *Y0;Y1~~0*Y1;Y2~~ 0 *Y2;Y3~~ 0 *Y3;Y4~~ 0*Y4; omega>0;omega2>0 ; omega3>0;omega4>0;
'
#Model fitting
fit <- cfa(FAIY, data=cbind(Y0,Y1,Y2,Y3,Y4));
PhiY<- parameterEstimates(fit)[20,5] #Trait factor variance estimate of Y
LambdaY<-rep(1,5);PsiY<- fitted(fit)$cov-PhiY;SigmaY<- cov(cbind(Y0,Y1,Y2,Y3,Y4))

FYres<-rep(0,N); #Store correlation-preserving predictor for Y
for(i in 1:N){
YY<-c(Y0[i],Y1[i], Y2[i], Y3[i], Y4[i])
FYres[i]<-PhiY^(2)* (1/sqrt(PhiY^(3)*t(LambdaY)%*%solve(SigmaY)%*%LambdaY))%*%t(LambdaY)%*% solve(SigmaY)%*%YY
}
FYres<-FYres-mean(FYres)
#Evalute the presence of improper solutions
IMPRO2<-sum(c(sign(parameterEstimates(fit)[c(20:24,30),5])) )+ ifelse(is.na(FYres[1]),0,1)+ ifelse(is.na(FYbart[1]),0,1) #8
```

#Step 1.1 Estimate model parameters for a variable L

```
#lavaan code
FAIL <- 'TraitL =~1*L0 +1*L1 +1*L2+1*L3;
```

```
dL0=~1*L0; dL1=~1*L1; dL2=~1*L2; dL3=~1*L3
dL1~beta*dL0; dL2~beta2*dL1; dL3~beta3*dL2;
TraitL~~0*dL0; TraitL~~0*dL1; TraitL~~0*dL2; TraitL~~0*dL3;
TraitL~~vy*TraitL;
L0~~ 0*L0; L1~~ 0*L1;L2~~ 0*L2;L3~~ 0*L3;omega>0
dL1~~omega*dL1; dL2~~omega2*dL2; dL3~~omega3*dL3;
;omega>0;omega2>0;omega3>0'
#Model fitting
fit <- cfa(FAIL, data=cbind(L0,L1,L2,L3))
PhiL<- parameterEstimates(fit)[16,5] #Trait factor variance estimate of L
LambdaL<-rep(1,4);PsiL<- fitted(fit)$cov-PhiL;SigmaL<-cov(cbind(L0,L1,L2,L3))

FLres<-rep(0,N) #Store correlation-preserving predictor for Y
for(i in 1:N){
LL<-c(L0[i], L1[i], L2[i], L3[i])
FLres[i]<-PhiL^(2)* (1/sqrt(PhiL^(3)*t(LambdaL)%*%solve(SigmaL)%*%LambdaL))%*%t(LambdaL)%*% solve(SigmaL)%*%LL
}
FLres<-FLres-mean(FLres);
#Evaluate the presence of improper solutions
IMPRO3<-sum(c(sign(parameterEstimates(fit)[c(16,21:24),5])) ) + ifelse(is.na(FLres[1]),0,1)+ ifelse(is.na(FLbart[1]),0,1) #7
```

#Step 1.1 Estimate model parameters for a variable A

```
#lavaan code
FAIA <- 'TraitA =~1*A0 +1*A1 +1*A2+1*A3
dA0=~1*A0; dA1=~1*A1; dA2=~1*A2; dA3=~1*A3;
dA1~beta*dA0; dA2~beta2*dA1; dA3~beta3*dA2
TraitA~~0*dA0; TraitA~~0*dA1; TraitA~~0*dA2; TraitA~~0*dA3;
TraitA~~vy*TraitA;
A0~~0*A0; A1~~ 0*A1;A2~~ 0*A2;A3~~ 0*A3
dA1~~omega*dA1; dA2~~omega2*dA2; dA3~~omega3*dA3;
;omega>0;omega2>0;omega3>0 '
#Model fitting
fit <- cfa(FAIA, data=cbind(A0,A1,A2,A3))
PhiA<- parameterEstimates(fit)[16,5] #Trait factor variance estimate of A
LambdaA<-rep(1,4);PsiA<- fitted(fit)$cov-PhiA;SigmaA<- cov(cbind(A0,A1,A2,A3))
FAres<-rep(0,N) #Store correlation-preserving predictor for A
for(i in 1:N){
AA<-c(A0[i], A1[i], A2[i], A3[i])
FAres[i]<-PhiA^(2)* (1/sqrt(PhiA^(3)*t(LambdaA)%*%solve(SigmaA)%*%LambdaA))%*%t(LambdaA)%*% solve(SigmaA)%*%AA
}
FAres<-FAres-mean(FAres)
#Evaluate the presence of improper solutions
IMPRO4<-sum(c(sign(parameterEstimates(fit)[c(16,21:24),5])) ) + ifelse(is.na(FAres[1]),0,1)+ ifelse(is.na(FAbart[1]),0,1) #7

#Continue if no improper solutions were found
if(IMPRO1+ IMPRO2+ IMPRO3+ IMPRO4>21){
```

#Step 1.2 Predict within-person variability scores

```
SigmaX<-var(cbind(Y0,Y1,Y2,Y3,Y4,A0,A1,A2,A3,L0,L1,L2,L3))
Imat<-matrix(rep(0,13*13),13,13) #Store trait (co)variances matrix
Imat[1:5,1:5]<-PhiY; Imat[6:9,6:9]<-PhiA; Imat[10:13,10:13]<-PhiL;
Imat[1:5,6:9]<-cov(FYres,FAres);Imat[6:9,1:5]<-cov(FYres,FAres);Imat[1:5,10:13]<-cov(FYres,FLres)
Imat[10:13,1:5]<-cov(FYres,FLres);Imat[6:9,10:13]<-cov(FAres,FLres);Imat[10:13,6:9]<-cov(FAres,FLres)
PsiX<-SigmaX-Imat

#Singular value decomposition
  UPsiX <- svd(PsiX)$u; VPsiX <- svd(PsiX)$v ;DPsiX <- diag(sqrt(svd(PsiX)$d))
  PsiXhalf<- UPsiX %*% DPsiX %*% t(VPsiX)

  USigmaX <- svd(SigmaX)$u; VSigmaX <- svd(SigmaX)$v; DSigmaX <- diag(sqrt(svd(SigmaX)$d))
  SigmaXhalf<- USigmaX %*% DSigmaX %*% t(VSigmaX)

            SIGX<-PsiX%*%PsiXhalf%*%solve(SigmaX)%*%PsiX%*%PsiXhalf

  USIGX <- svd(SIGX)$u;VSIGX <- svd(SIGX)$v; DSIGX <- diag(sqrt(svd(SIGX)$d))
  SIGXhalf<- USIGX %*% DSIGX %*% t(VSIGX)

#Calculate weights
    WWX<-PsiXhalf%*% solve(SIGXhalf)%*% PsiX%*%PsiXhalf%*%solve(SigmaX)

#Calculate within-person variability scores
FYwith2<-matrix(rep(0,N*13),N,13);for(i in 1:N){
XX<-c(Y0[i],Y1[i], Y2[i], Y3[i], Y4[i], A0[i],A1[i], A2[i], A3[i], L0[i],L1[i], L2[i], L3[i])
FYwith2[i,]<-WWX%*%XX
}
```

#Step 2 Estimate Causal Parameters by SNMM

#True score condition

```
Y0<-   dY0
Y1<-   dY1
Y2<-   dY2
Y3<-   dY3
Y4<-    dY4
L0<-dL0; L1<-dL1; L2<-dL2; L3<-dL3; A0<-dA0; A1<-dA1; A2<-dA2; A3<-dA3

Y3sq<-Y3^2;L3sq<-L3^2;A2sq<-A2^2; Y2sq<-Y2^2;L2sq<-L2^2;A1sq<-A1^2; Y1sq<-Y1^2;L1sq<-L1^2;A0sq<-A0^2; Y0sq<-Y0^2;L0sq<-L0^2;
Y3L3<-Y3*L3; Y2L2<-Y2*L2; Y1L1<-Y1*L1; Y0L0<-Y0*L0;
Y3A3<-Y3*A3; Y2A2<-Y2*A2; Y1A1<-Y1*A1; Y0A0<-Y0*A0;
L3A3<-L3*A3; L2A2<-L2*A2; L1A1<-L1*A1; L0A0<-L0*A0;
#Estimate P(Y|A,L)
PredY43<-lm(Y4~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredY42<-lm(Y4~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY41<-lm(Y4~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY40<-lm(Y4~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY32<-lm(Y3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY31<-lm(Y3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY30<-lm(Y3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY21<-lm(Y2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY20<-lm(Y2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY10<-lm(Y1~ Y0+L0+Y0sq+L0sq +Y0L0)
#Estimate P(A|A,L)
PredA33<-lm(A3~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredA32<-lm(A3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA31<-lm(A3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA30<-lm(A3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA22<-lm(A2~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA21<-lm(A2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA20<-lm(A2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA11<-lm(A1~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA10<-lm(A1~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA00<-lm(A0~ Y0+L0+Y0sq+L0sq+Y0L0)

DATA3<-data.frame(cbind(Y3,L3,A2, Y3sq,L3sq ,A2sq,Y3L3));DATA2<-data.frame(cbind(Y2,L2,A1, Y2sq,L2sq ,A1sq,Y2L2))
DATA1<-data.frame(cbind(Y1,L1,A0, Y1sq,L1sq ,A0sq,Y1L1));DATA0<-data.frame(cbind(Y0,L0, Y0sq,L0sq ,Y0L0))
#Set initial values
psi1<-0.5; psi2<-0.3; psi3<-0.5; psi4<-0.3; psi5<-0.5; psi6<-0.5; psi7<-0.5; psi8<-0.3; psi9<-0.5; psi10<-0.5

V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
#Solve estimating equations via the Newton-Raphson Method
CHECK<-0; ITER1<-0; while(CHECK<1 && ITER1<200){
#First-order differentiation
dI<-rep(0,10); ITER1<-ITER1+1
dI[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)*(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
)
dI[2]<-sum(
(A2-predict(PredA22,DATA2))*(1/V2)*(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
)
dI[3]<-sum(
(A1-predict(PredA11,DATA1))*(1/V3)*(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-
predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
)
dI[4]<-sum(
(A0-predict(PredA00,DATA0))*(1/V4)*(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-
predict(PredA20,DATA0))+psi3*(A1-predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
)

dI[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)*(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
)
dI[6]<-sum(
```

```
(A1-predict(PredA11,DATA1))*(1/V6)*(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
)
dI[7]<-sum(
(A0-predict(PredA00,DATA0))*(1/V7)*(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-
predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
)
dI[8]<-sum(
(A1-predict(PredA11,DATA1))*(1/V8)*(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
)
dI[9]<-sum(
(A0-predict(PredA00,DATA0))*(1/V9)*(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
)
dI[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)*( Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
)

#Second-order differentiation
dII<-rep(0,5)
dII[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)* (A3-predict(PredA33,DATA3))
)
dII[2]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V2)* (A2-predict(PredA22,DATA2))
)
dII[3]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V3)* (A1-predict(PredA11,DATA1))
)
dII[4]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V4)* (A0-predict(PredA00,DATA0))
)
dII[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)* (A2-predict(PredA22,DATA2))
)
dII[6]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V6)* (A1-predict(PredA11,DATA1))
)
dII[7]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V7)* (A0-predict(PredA00,DATA0))
)
dII[8]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V8)* (A1-predict(PredA11,DATA1))
)
dII[9]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V9)* (A0-predict(PredA00,DATA0))
)
dII[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)* (A0-predict(PredA00,DATA0))
)

#Update parameter values
PSI<- c(psi1,psi2,psi3,psi4,psi5,psi6,psi7,psi8,psi9,psi10);NEW<-PSI+diag(1/dII)%*%dI
#Check the convergence criterion
CHECK<-ifelse(max(abs(PSI-NEW))<0.0001,1,0)
psi1<-NEW[1] ; psi2<-NEW[2]; psi3<-NEW[3] ; psi4<-NEW[4] ; psi5<-NEW[5]; psi6<-NEW[6]
psi7<-NEW[7] ; psi8<-NEW[8]; psi9<-NEW[9] ; psi10<-NEW[10]
V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
};
#Store estimates and SEs
PSI1<-PSI;SE1<-sqrt(1/dII)
```

```
#Observed score (no centering) condition
Y0<-   YY0-mean(YY0)
Y1<-   YY1-mean(YY1)
Y2<-   YY2-mean(YY2)
Y3<-   YY3-mean(YY3)
Y4<-   YY4-mean(YY4)
L0<-LL0-mean(LL0);  L1<-LL1-mean(LL1);  L2<-LL2-mean(LL2);  L3<-LL3-mean(LL3);  A0<-AA0-mean(AA0);  A1<-AA1-mean(AA1);  A2<-AA2-
mean(AA2); A3<-AA3-mean(AA3)


Y3sq<-Y3^2;L3sq<-L3^2;A2sq<-A2^2; Y2sq<-Y2^2;L2sq<-L2^2;A1sq<-A1^2; Y1sq<-Y1^2;L1sq<-L1^2;A0sq<-A0^2; Y0sq<-Y0^2;L0sq<-L0^2;
Y3L3<-Y3*L3; Y2L2<-Y2*L2; Y1L1<-Y1*L1; Y0L0<-Y0*L0;
Y3A3<-Y3*A3; Y2A2<-Y2*A2; Y1A1<-Y1*A1; Y0A0<-Y0*A0;
L3A3<-L3*A3; L2A2<-L2*A2; L1A1<-L1*A1; L0A0<-L0*A0;
#Estimate P(Y|A,L)
PredY43<-lm(Y4~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredY42<-lm(Y4~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY41<-lm(Y4~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY40<-lm(Y4~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY32<-lm(Y3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY31<-lm(Y3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY30<-lm(Y3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY21<-lm(Y2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY20<-lm(Y2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY10<-lm(Y1~ Y0+L0+Y0sq+L0sq +Y0L0)
#Estimate P(A|A,L)
PredA33<-lm(A3~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredA32<-lm(A3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA31<-lm(A3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA30<-lm(A3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA22<-lm(A2~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA21<-lm(A2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA20<-lm(A2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA11<-lm(A1~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA10<-lm(A1~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA00<-lm(A0~ Y0+L0+Y0sq+L0sq+Y0L0)

DATA3<-data.frame(cbind(Y3,L3,A2, Y3sq,L3sq ,A2sq,Y3L3));DATA2<-data.frame(cbind(Y2,L2,A1, Y2sq,L2sq ,A1sq,Y2L2))
DATA1<-data.frame(cbind(Y1,L1,A0, Y1sq,L1sq ,A0sq,Y1L1));DATA0<-data.frame(cbind(Y0,L0, Y0sq,L0sq ,Y0L0))
#Set initial values
psi1<-0.5; psi2<-0.3; psi3<-0.5; psi4<-0.3; psi5<-0.5; psi6<-0.5; psi7<-0.5; psi8<-0.3; psi9<-0.5; psi10<-0.5

V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
#Solve estimating equations via the Newton-Raphson Method
CHECK<-0; ITER2<-0; while(CHECK<1 && ITER2<200){
#First-order differentiation
dI<-rep(0,10); ITER2<-ITER2+1
dI[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)*(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
)
dI[2]<-sum(
(A2-predict(PredA22,DATA2))*(1/V2)*(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
)
dI[3]<-sum(
(A1-predict(PredA11,DATA1))*(1/V3)*(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-
predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
)
dI[4]<-sum(
(A0-predict(PredA00,DATA0))*(1/V4)*(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-
predict(PredA20,DATA0))+psi3*(A1-predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
)

dI[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)*(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
)
dI[6]<-sum(
```

```
(A1-predict(PredA11,DATA1))*(1/V6)*(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
)
dI[7]<-sum(
(A0-predict(PredA00,DATA0))*(1/V7)*(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-
predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
)
dI[8]<-sum(
(A1-predict(PredA11,DATA1))*(1/V8)*(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
)
dI[9]<-sum(
(A0-predict(PredA00,DATA0))*(1/V9)*(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
)
dI[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)*( Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
)

#Second-order differentiation
dII<-rep(0,5)
dII[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)* (A3-predict(PredA33,DATA3))
)
dII[2]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V2)* (A2-predict(PredA22,DATA2))
)
dII[3]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V3)* (A1-predict(PredA11,DATA1))
)
dII[4]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V4)* (A0-predict(PredA00,DATA0))
)
dII[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)* (A2-predict(PredA22,DATA2))
)
dII[6]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V6)* (A1-predict(PredA11,DATA1))
)
dII[7]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V7)* (A0-predict(PredA00,DATA0))
)
dII[8]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V8)* (A1-predict(PredA11,DATA1))
)
dII[9]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V9)* (A0-predict(PredA00,DATA0))
)
dII[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)* (A0-predict(PredA00,DATA0))
)

#Update parameter values
PSI<- c(psi1,psi2,psi3,psi4,psi5,psi6,psi7,psi8,psi9,psi10);NEW<-PSI+diag(1/dII)%*%dI
#Check the convergence criterion
CHECK<-ifelse(max(abs(PSI-NEW))<0.0001,1,0)
psi1<-NEW[1] ; psi2<-NEW[2]; psi3<-NEW[3] ; psi4<-NEW[4] ; psi5<-NEW[5]; psi6<-NEW[6]
psi7<-NEW[7] ; psi8<-NEW[8]; psi9<-NEW[9] ; psi10<-NEW[10]
V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
};
#Store estimates and SEs
PSI2<-PSI;SE2<-sqrt(1/dII)
```

```
#Observed-mean centering condition
MY<-rowMeans(cbind(YY0,YY1,YY2,YY3)); ML<-rowMeans(cbind(LL0,LL1,LL2,LL3)); MA<-rowMeans(cbind(AA0,AA1,AA2,AA3))
Y0<-     (YY0-mean(YY0)-MY)
Y1<-     (YY1-mean(YY1)-MY)
Y2<-     (YY2-mean(YY2)-MY)
Y3<-     (YY3-mean(YY3)-MY)
Y4<-     (YY4-mean(YY4)-MY)
L0<-LL0-mean(LL0)-ML; L1<-LL1-mean(LL1)-ML; L2<-LL2-mean(LL2)-ML; L3<-LL3-mean(LL3)-ML;
A0<-AA0-mean(AA0)-MA; A1<-AA1-mean(AA1)-MA; A2<-AA2-mean(AA2)-MA; A3<-AA3-mean(AA3)-MA

Y3sq<-Y3^2;L3sq<-L3^2;A2sq<-A2^2; Y2sq<-Y2^2;L2sq<-L2^2;A1sq<-A1^2; Y1sq<-Y1^2;L1sq<-L1^2;A0sq<-A0^2; Y0sq<-Y0^2;L0sq<-L0^2;
Y3L3<-Y3*L3; Y2L2<-Y2*L2; Y1L1<-Y1*L1; Y0L0<-Y0*L0;
Y3A3<-Y3*A3; Y2A2<-Y2*A2; Y1A1<-Y1*A1; Y0A0<-Y0*A0;
L3A3<-L3*A3; L2A2<-L2*A2; L1A1<-L1*A1; L0A0<-L0*A0;
#Estimate P(Y|A,L)
PredY43<-lm(Y4~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredY42<-lm(Y4~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY41<-lm(Y4~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY40<-lm(Y4~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY32<-lm(Y3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY31<-lm(Y3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY30<-lm(Y3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY21<-lm(Y2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY20<-lm(Y2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY10<-lm(Y1~ Y0+L0+Y0sq+L0sq +Y0L0)
#Estimate P(A|A,L)
PredA33<-lm(A3~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredA32<-lm(A3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA31<-lm(A3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA30<-lm(A3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA22<-lm(A2~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA21<-lm(A2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA20<-lm(A2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA11<-lm(A1~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA10<-lm(A1~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA00<-lm(A0~ Y0+L0+Y0sq+L0sq+Y0L0)

DATA3<-data.frame(cbind(Y3,L3,A2, Y3sq,L3sq ,A2sq,Y3L3));DATA2<-data.frame(cbind(Y2,L2,A1, Y2sq,L2sq ,A1sq,Y2L2))
DATA1<-data.frame(cbind(Y1,L1,A0, Y1sq,L1sq ,A0sq,Y1L1));DATA0<-data.frame(cbind(Y0,L0, Y0sq,L0sq ,Y0L0))
#Set initial values
psi1<-0.5; psi2<-0.3; psi3<-0.5; psi4<-0.3; psi5<-0.5; psi6<-0.5; psi7<-0.5; psi8<-0.3; psi9<-0.5; psi10<-0.5

V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
#Solve estimating equations via the Newton-Raphson Method
CHECK<-0; ITER3<-0; while(CHECK<1 && ITER3<200){
#First-order differentiation
dI<-rep(0,10); ITER3<-ITER3+1
dI[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)*(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
)
dI[2]<-sum(
(A2-predict(PredA22,DATA2))*(1/V2)*(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
)
dI[3]<-sum(
(A1-predict(PredA11,DATA1))*(1/V3)*(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-
predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
)
dI[4]<-sum(
(A0-predict(PredA00,DATA0))*(1/V4)*(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-
predict(PredA20,DATA0))+psi3*(A1-predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
)

dI[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)*(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
)
dI[6]<-sum(
```

```
(A1-predict(PredA11,DATA1))*(1/V6)*(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
)
dI[7]<-sum(
(A0-predict(PredA00,DATA0))*(1/V7)*(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-
predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
)
dI[8]<-sum(
(A1-predict(PredA11,DATA1))*(1/V8)*(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
)
dI[9]<-sum(
(A0-predict(PredA00,DATA0))*(1/V9)*(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
)
dI[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)*( Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
)

#Second-order differentiation
dII<-rep(0,5)
dII[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)* (A3-predict(PredA33,DATA3))
)
dII[2]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V2)* (A2-predict(PredA22,DATA2))
)
dII[3]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V3)* (A1-predict(PredA11,DATA1))
)
dII[4]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V4)* (A0-predict(PredA00,DATA0))
)
dII[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)* (A2-predict(PredA22,DATA2))
)
dII[6]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V6)* (A1-predict(PredA11,DATA1))
)
dII[7]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V7)* (A0-predict(PredA00,DATA0))
)
dII[8]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V8)* (A1-predict(PredA11,DATA1))
)
dII[9]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V9)* (A0-predict(PredA00,DATA0))
)
dII[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)* (A0-predict(PredA00,DATA0))
)

#Update parameter values
PSI<- c(psi1,psi2,psi3,psi4,psi5,psi6,psi7,psi8,psi9,psi10);NEW<-PSI+diag(1/dII)%*%dI
#Check the convergence criterion
CHECK<-ifelse(max(abs(PSI-NEW))<0.0001,1,0)
psi1<-NEW[1] ; psi2<-NEW[2]; psi3<-NEW[3] ; psi4<-NEW[4] ; psi5<-NEW[5]; psi6<-NEW[6]
psi7<-NEW[7] ; psi8<-NEW[8]; psi9<-NEW[9] ; psi10<-NEW[10]
V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
};
#Store estimates and SEs
PSI3<-PSI;SE3<-sqrt(1/dII)
```

```
#Proposed method
Y0<-    FYwith2[,1]
Y1<-    FYwith2[,2]
Y2<-    FYwith2[,3]
Y3<-    FYwith2[,4]
Y4<-    FYwith2[,5]
L0<-    FYwith2[,10]
L1<-    FYwith2[,11]
L2<-    FYwith2[,12]
L3<-    FYwith2[,13]
A0<-    FYwith2[,6]
A1<-    FYwith2[,7]
A2<-    FYwith2[,8]
A3<-    FYwith2[,9]

Y3sq<-Y3^2;L3sq<-L3^2;A2sq<-A2^2; Y2sq<-Y2^2;L2sq<-L2^2;A1sq<-A1^2; Y1sq<-Y1^2;L1sq<-L1^2;A0sq<-A0^2; Y0sq<-Y0^2;L0sq<-L0^2;
Y3L3<-Y3*L3; Y2L2<-Y2*L2; Y1L1<-Y1*L1; Y0L0<-Y0*L0;
Y3A3<-Y3*A3; Y2A2<-Y2*A2; Y1A1<-Y1*A1; Y0A0<-Y0*A0;
L3A3<-L3*A3; L2A2<-L2*A2; L1A1<-L1*A1; L0A0<-L0*A0;
#Estimate P(Y|A,L)
PredY43<-lm(Y4~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredY42<-lm(Y4~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY41<-lm(Y4~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY40<-lm(Y4~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY32<-lm(Y3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY31<-lm(Y3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY30<-lm(Y3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY21<-lm(Y2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY20<-lm(Y2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY10<-lm(Y1~ Y0+L0+Y0sq+L0sq +Y0L0)
#Estimate P(A|A,L)
PredA33<-lm(A3~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredA32<-lm(A3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA31<-lm(A3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA30<-lm(A3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA22<-lm(A2~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA21<-lm(A2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA20<-lm(A2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA11<-lm(A1~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA10<-lm(A1~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA00<-lm(A0~ Y0+L0+Y0sq+L0sq+Y0L0)

DATA3<-data.frame(cbind(Y3,L3,A2, Y3sq,L3sq ,A2sq,Y3L3));DATA2<-data.frame(cbind(Y2,L2,A1, Y2sq,L2sq ,A1sq,Y2L2))
DATA1<-data.frame(cbind(Y1,L1,A0, Y1sq,L1sq ,A0sq,Y1L1));DATA0<-data.frame(cbind(Y0,L0, Y0sq,L0sq ,Y0L0))
#Set initial values
psi1<-0.5; psi2<-0.3; psi3<-0.5; psi4<-0.3; psi5<-0.5; psi6<-0.5; psi7<-0.5; psi8<-0.3; psi9<-0.5; psi10<-0.5

V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
#Solve estimating equations via the Newton-Raphson Method
CHECK<-0; ITER4<-0; while(CHECK<1 && ITER4<200){
#First-order differentiation
dI<-rep(0,10); ITER4<-ITER4+1
dI[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)*(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
)
dI[2]<-sum(
(A2-predict(PredA22,DATA2))*(1/V2)*(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
)
dI[3]<-sum(
(A1-predict(PredA11,DATA1))*(1/V3)*(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-
predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
)
dI[4]<-sum(
(A0-predict(PredA00,DATA0))*(1/V4)*(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-
predict(PredA20,DATA0))+psi3*(A1-predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
)
```

```
dI[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)*(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
)
dI[6]<-sum(
(A1-predict(PredA11,DATA1))*(1/V6)*(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
)
dI[7]<-sum(
(A0-predict(PredA00,DATA0))*(1/V7)*(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-
predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
)
dI[8]<-sum(
(A1-predict(PredA11,DATA1))*(1/V8)*(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
)
dI[9]<-sum(
(A0-predict(PredA00,DATA0))*(1/V9)*(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
)
dI[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)*( Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
)
#Second-order differentiation
dII<-rep(0,5)
dII[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)* (A3-predict(PredA33,DATA3))
)
dII[2]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V2)* (A2-predict(PredA22,DATA2))
)
dII[3]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V3)* (A1-predict(PredA11,DATA1))
)
dII[4]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V4)* (A0-predict(PredA00,DATA0))
)
dII[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)* (A2-predict(PredA22,DATA2))
)
dII[6]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V6)* (A1-predict(PredA11,DATA1))
)
dII[7]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V7)* (A0-predict(PredA00,DATA0))
)
dII[8]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V8)* (A1-predict(PredA11,DATA1))
)
dII[9]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V9)* (A0-predict(PredA00,DATA0))
)
dII[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)* (A0-predict(PredA00,DATA0))
)

#Update parameter values
PSI<- c(psi1,psi2,psi3,psi4,psi5,psi6,psi7,psi8,psi9,psi10);NEW<-PSI+diag(1/dII)%*%dI
#Check the convergence criterion
CHECK<-ifelse(max(abs(PSI-NEW))<0.0001,1,0)
psi1<-NEW[1] ; psi2<-NEW[2]; psi3<-NEW[3] ; psi4<-NEW[4] ; psi5<-NEW[5]; psi6<-NEW[6]
psi7<-NEW[7] ; psi8<-NEW[8]; psi9<-NEW[9] ; psi10<-NEW[10]
V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
};
#Store estimates and SEs
PSI4<-PSI;SE4<-sqrt(1/dII)
```

```
#(Univariate) correlation-preserving factor score centering condition
Y0<-     (YY0-mean(YY0)-FYres)
Y1<-     (YY1-mean(YY1)-FYres)
Y2<-     (YY2-mean(YY2)-FYres)
Y3<-     (YY3-mean(YY3)-FYres)
Y4<-     (YY4-mean(YY4)-FYres)
L0<-     (LL0-mean(LL0)-FLres)
L1<-     (LL1-mean(LL1)-FLres)
L2<-     (LL2-mean(LL2)-FLres)
L3<-       (LL3-mean(LL3)-FLres)
A0<-    (AA0-mean(AA0)-FAres)
A1<-     (AA1-mean(AA1)-FAres)
A2<-     (AA2-mean(AA2)-FAres)
A3<-     (AA3-mean(AA3)-FAres)

Y3sq<-Y3^2;L3sq<-L3^2;A2sq<-A2^2; Y2sq<-Y2^2;L2sq<-L2^2;A1sq<-A1^2; Y1sq<-Y1^2;L1sq<-L1^2;A0sq<-A0^2; Y0sq<-Y0^2;L0sq<-L0^2;
Y3L3<-Y3*L3; Y2L2<-Y2*L2; Y1L1<-Y1*L1; Y0L0<-Y0*L0;
Y3A3<-Y3*A3; Y2A2<-Y2*A2; Y1A1<-Y1*A1; Y0A0<-Y0*A0;
L3A3<-L3*A3; L2A2<-L2*A2; L1A1<-L1*A1; L0A0<-L0*A0;
#Estimate P(Y|A,L)
PredY43<-lm(Y4~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredY42<-lm(Y4~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY41<-lm(Y4~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY40<-lm(Y4~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY32<-lm(Y3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredY31<-lm(Y3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY30<-lm(Y3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY21<-lm(Y2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredY20<-lm(Y2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredY10<-lm(Y1~ Y0+L0+Y0sq+L0sq +Y0L0)
#Estimate P(A|A,L)
PredA33<-lm(A3~ Y3+L3+A2+ Y3sq+L3sq +A2sq+Y3L3)
PredA32<-lm(A3~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA31<-lm(A3~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA30<-lm(A3~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA22<-lm(A2~ Y2+L2+A1+ Y2sq+L2sq +A1sq+Y2L2)
PredA21<-lm(A2~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA20<-lm(A2~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA11<-lm(A1~ Y1+L1+A0+ Y1sq+L1sq +A0sq+Y1L1)
PredA10<-lm(A1~ Y0+L0+Y0sq+L0sq+Y0L0)
PredA00<-lm(A0~ Y0+L0+Y0sq+L0sq+Y0L0)

DATA3<-data.frame(cbind(Y3,L3,A2, Y3sq,L3sq ,A2sq,Y3L3));DATA2<-data.frame(cbind(Y2,L2,A1, Y2sq,L2sq ,A1sq,Y2L2))
DATA1<-data.frame(cbind(Y1,L1,A0, Y1sq,L1sq ,A0sq,Y1L1));DATA0<-data.frame(cbind(Y0,L0, Y0sq,L0sq ,Y0L0))
#Set initial values
psi1<-0.5; psi2<-0.3; psi3<-0.5; psi4<-0.3; psi5<-0.5; psi6<-0.5; psi7<-0.5; psi8<-0.3; psi9<-0.5; psi10<-0.5

V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
#Solve estimating equations via the Newton-Raphson Method
CHECK<-0; ITER5<-0; while(CHECK<1 && ITER5<200){
#First-order differentiation
dI<-rep(0,10); ITER5<-ITER5+1
dI[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)*(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
)
dI[2]<-sum(
(A2-predict(PredA22,DATA2))*(1/V2)*(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
)
dI[3]<-sum(
(A1-predict(PredA11,DATA1))*(1/V3)*(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-
predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
)
dI[4]<-sum(
(A0-predict(PredA00,DATA0))*(1/V4)*(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-
predict(PredA20,DATA0))+psi3*(A1-predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
)
```

```
dI[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)*(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
)
dI[6]<-sum(
(A1-predict(PredA11,DATA1))*(1/V6)*(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
)
dI[7]<-sum(
(A0-predict(PredA00,DATA0))*(1/V7)*(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-
predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
)
dI[8]<-sum(
(A1-predict(PredA11,DATA1))*(1/V8)*(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
)
dI[9]<-sum(
(A0-predict(PredA00,DATA0))*(1/V9)*(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
)
dI[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)*( Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
)
dII<-rep(0,5)
#Second-order differentiation
dII[1]<-sum(
(A3-predict(PredA33,DATA3)) *(1/V1)* (A3-predict(PredA33,DATA3))
)
dII[2]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V2)* (A2-predict(PredA22,DATA2))
)
dII[3]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V3)* (A1-predict(PredA11,DATA1))
)
dII[4]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V4)* (A0-predict(PredA00,DATA0))
)
dII[5]<-sum(
(A2-predict(PredA22,DATA2)) *(1/V5)* (A2-predict(PredA22,DATA2))
)
dII[6]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V6)* (A1-predict(PredA11,DATA1))
)
dII[7]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V7)* (A0-predict(PredA00,DATA0))
)
dII[8]<-sum(
(A1-predict(PredA11,DATA1)) *(1/V8)* (A1-predict(PredA11,DATA1))
)
dII[9]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V9)* (A0-predict(PredA00,DATA0))
)
dII[10]<-sum(
(A0-predict(PredA00,DATA0)) *(1/V10)* (A0-predict(PredA00,DATA0))
)

#Update parameter values
PSI<- c(psi1,psi2,psi3,psi4,psi5,psi6,psi7,psi8,psi9,psi10);NEW<-PSI+diag(1/dII)%*%dI
#Check the convergence criterion
CHECK<-ifelse(max(abs(PSI-NEW))<0.0001,1,0)
psi1<-NEW[1] ; psi2<-NEW[2]; psi3<-NEW[3] ; psi4<-NEW[4] ; psi5<-NEW[5]; psi6<-NEW[6]
psi7<-NEW[7] ; psi8<-NEW[8]; psi9<-NEW[9] ; psi10<-NEW[10]
V1<-var(Y4-predict(PredY43,DATA3)-(psi1*(A3-predict(PredA33,DATA3))))
V2<-var(Y4-predict(PredY42,DATA2)-(psi1*(A3-predict(PredA32,DATA2))+psi2*(A2-predict(PredA22,DATA2))))
V3<-var(Y4-predict(PredY41,DATA1)-(psi1*(A3-predict(PredA31,DATA1))+psi2*(A2-predict(PredA21,DATA1))+psi3*(A1-predict(PredA11,DATA1))))
V4<-var(Y4-predict(PredY40,DATA0)-(psi1*(A3-predict(PredA30,DATA0))+psi2*(A2-predict(PredA20,DATA0))+psi3*(A1-
predict(PredA10,DATA0))+psi4*(A0-predict(PredA00,DATA0))))
V5<-var(Y3-predict(PredY32,DATA2)-(psi5*(A2-predict(PredA22,DATA2))))
V6<-var(Y3-predict(PredY31,DATA1)-(psi5*(A2-predict(PredA21,DATA1))+psi6*(A1-predict(PredA11,DATA1))))
V7<-var(Y3-predict(PredY30,DATA0)-(psi5*(A2-predict(PredA20,DATA0))+psi6*(A1-predict(PredA10,DATA0))+psi7*(A0-predict(PredA00,DATA0))))
V8<-var(Y2-predict(PredY21,DATA1)-(psi8*(A1-predict(PredA11,DATA1))))
V9<-var(Y2-predict(PredY20,DATA0)-(psi8*(A1-predict(PredA10,DATA0))+psi9*(A0-predict(PredA00,DATA0))))
V10<-var(Y1-predict(PredY10,DATA0)-(psi10*(A0-predict(PredA00,DATA0))))
};
#Store estimates and SEs
PSI5<-PSI;SE5<-sqrt(1/dII)
```

```
#Step 2 Estimate Causal Parameters by MSM
#True score condition
ZY_1<- dY0; ZY_2<- dY1; ZY_3<- dY2; ZY_4<- dY3
ZA_1<-dA0; ZA_2<-dA1; ZA_3<-dA2; ZA_4<-dA3;
ZL_1<-dL0; ZL_2<-dL1; ZL_3<-dL2; ZL_4<-dL3
ZLL_2<-dA0; ZLL_3<-dA1; ZLL_4<-dA2
ZZY_1<- dY1;ZZY_2<- dY2;ZZY_3<- dY3;ZZY_4<- dY4
ZZA_1<-dA0; ZZA_2<-dA1; ZZA_3<-dA2; ZZA_4<-dA3;

id<-c(1:N);data1 <- data.frame(id,ZY_1,ZA_1,ZL_1)
data_long<-reshape(data1, varying=c("ZY_1", "ZA_1", "ZL_1"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A0
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ 1,
   denominator = ~ ZL+ZY,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw1 = c(w$ipw.weights)

iptww<-iptw1; data2 <- data.frame(id,ZZY_1,ZZA_1,iptww)
#Estimate causal effect b11 (for outcome at k=1)
gee.iptw <- lm(ZZY_1~ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M11est<-out$coefficients[2,1]

data1 <- data.frame(id,ZY_2 ,ZA_2,ZL_2,ZLL_2)
data_long<-reshape(data1, varying=c("ZY_2","ZA_2","ZL_2","ZLL_2"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A1
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw2 = c(w$ipw.weights)

iptww<-iptw1*iptw2; data2 <- data.frame(id,ZZY_2,ZZA_1,ZZA_2,iptww)
#Estimate causal effects b21 and b22 (for outcome at k=2)
gee.iptw <- lm(ZZY_2~ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M21est<-out$coefficients[2,1]; M22est<-out$coefficients[3,1];


data1 <- data.frame(id,ZY_3 ,ZA_3,ZL_3,ZLL_3)
data_long<-reshape(data1, varying=c("ZY_3","ZA_3","ZL_3","ZLL_3"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A2
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw3= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3; data2 <- data.frame(id,ZZY_3, ZZA_1,ZZA_2,ZZA_3,iptww)
#Estimate causal effects b31, b32, and b33 (for outcome at k=3)
gee.iptw <- lm(ZZY_3~ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M31est<-out$coefficients[2,1]; M32est<-out$coefficients[3,1];M33est<-out$coefficients[4,1];


data1 <- data.frame(id,ZY_4 ,ZA_4,ZL_4,ZLL_4)
data_long<-reshape(data1, varying=c("ZY_4","ZA_4","ZL_4","ZLL_4"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A3
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw4= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3*iptw4; data2 <- data.frame(id,ZZY_4,ZZA_1,ZZA_2,ZZA_3,ZZA_4,iptww)
#Estimate causal effects b41, b42, b43, and b44 (for outcome at k=4)
gee.iptw <- lm(ZZY_4~ZZA_4+ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M41est<-out$coefficients[2,1]; M42est<-out$coefficients[3,1];M43est<-out$coefficients[4,1] ;M44est<-out$coefficients[5,1]

#Store estimation results
MPSI1<-c(M41est,M42est,M43est,M44est,M31est,M32est,M33est,M21est,M22est,M11est)
```

```
#Observed score (no centering) condition
ZY_1<-      YY0-mean(YY0);ZY_2<-      YY1-mean(YY1);ZY_3<-      YY2-mean(YY2);ZY_4<-      YY3-mean(YY3)
ZA_1<-AA0-mean(AA0); ZA_2<-AA1-mean(AA1); ZA_3<- AA2-mean(AA2); ZA_4<- AA3-mean(AA3);
ZL_1<- LL0-mean(LL0); ZL_2<- LL1-mean(LL1); ZL_3<- LL2-mean(LL2); ZL_4<- LL3-mean(LL3)
ZLL_2<- AA0-mean(AA0); ZLL_3<- AA1-mean(AA1); ZLL_4<- AA2-mean(AA2)
ZZY_1<- YY1-mean(YY1);ZZY_2<- YY2-mean(YY2);ZZY_3<- YY3-mean(YY3);ZZY_4<- YY4-mean(YY4)
ZZA_1<- AA0-mean(AA0); ZZA_2<- AA1-mean(AA1); ZZA_3<- AA2-mean(AA2); ZZA_4<- AA3-mean(AA3);

id<-c(1:N);data1 <- data.frame(id,ZY_1,ZA_1,ZL_1)
data_long<-reshape(data1, varying=c("ZY_1", "ZA_1", "ZL_1"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A0
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ 1,
   denominator = ~ ZL+ZY,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw1 = c(w$ipw.weights)

iptww<-iptw1; data2 <- data.frame(id,ZZY_1,ZZA_1,iptww)
#Estimate causal effect b11 (for outcome at k=1)
gee.iptw <- lm(ZZY_1~ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M11est<-out$coefficients[2,1]

data1 <- data.frame(id,ZY_2 ,ZA_2,ZL_2,ZLL_2)
data_long<-reshape(data1, varying=c("ZY_2","ZA_2","ZL_2","ZLL_2"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A1
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw2 = c(w$ipw.weights)

iptww<-iptw1*iptw2; data2 <- data.frame(id,ZZY_2,ZZA_1,ZZA_2,iptww)
#Estimate causal effects b21 and b22 (for outcome at k=2)
gee.iptw <- lm(ZZY_2~ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M21est<-out$coefficients[2,1]; M22est<-out$coefficients[3,1];


data1 <- data.frame(id,ZY_3 ,ZA_3,ZL_3,ZLL_3)
data_long<-reshape(data1, varying=c("ZY_3","ZA_3","ZL_3","ZLL_3"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A2
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw3= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3; data2 <- data.frame(id,ZZY_3, ZZA_1,ZZA_2,ZZA_3,iptww)
#Estimate causal effects b31, b32, and b33 (for outcome at k=3)
gee.iptw <- lm(ZZY_3~ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M31est<-out$coefficients[2,1]; M32est<-out$coefficients[3,1];M33est<-out$coefficients[4,1];


data1 <- data.frame(id,ZY_4 ,ZA_4,ZL_4,ZLL_4)
data_long<-reshape(data1, varying=c("ZY_4","ZA_4","ZL_4","ZLL_4"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A3
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw4= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3*iptw4; data2 <- data.frame(id,ZZY_4,ZZA_1,ZZA_2,ZZA_3,ZZA_4,iptww)
#Estimate causal effect b41, b42, b43, and b44 (for outcome at k=4)
gee.iptw <- lm(ZZY_4~ZZA_4+ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M41est<-out$coefficients[2,1]; M42est<-out$coefficients[3,1];M43est<-out$coefficients[4,1] ;M44est<-out$coefficients[5,1]

#Store estimation results
MPSI2<-c(M41est,M42est,M43est,M44est,M31est,M32est,M33est,M21est,M22est,M11est)
```

```
#Observed-mean centering condition
MY<-rowMeans(cbind(YY0,YY1,YY2,YY3,YY4)); ML<-rowMeans(cbind(LL0,LL1,LL2,LL3)); MA<-rowMeans(cbind(AA0,AA1,AA2,AA3))
ZY_1<-  YY0-mean(YY0)-MY;ZY_2<-  YY1-mean(YY1) -MY;ZY_3<-  YY2-mean(YY2) -MY;ZY_4<-  YY3-mean(YY3) -MY
ZA_1<-AA0-mean(AA0)-MA; ZA_2<-AA1-mean(AA1)-MA; ZA_3<- AA2-mean(AA2)-MA; ZA_4<- AA3-mean(AA3)-MA;
ZL_1<- LL0-mean(LL0)-ML; ZL_2<- LL1-mean(LL1)-ML; ZL_3<- LL2-mean(LL2)-ML; ZL_4<- LL3-mean(LL3)-ML
ZLL_2<- AA0-mean(AA0)-MA; ZLL_3<- AA1-mean(AA1)-MA; ZLL_4<- AA2-mean(AA2)-MA
ZZY_1<- YY1-mean(YY1) -MY;ZZY_2<- YY2-mean(YY2) -MY;ZZY_3<- YY3-mean(YY3) -MY;ZZY_4<- YY4-mean(YY4) -MY
ZZA_1<- AA0-mean(AA0)-MA; ZZA_2<- AA1-mean(AA1)-MA; ZZA_3<- AA2-mean(AA2)-MA; ZZA_4<- AA3-mean(AA3)-MA;

id<-c(1:N);data1 <- data.frame(id,ZY_1,ZA_1,ZL_1)
data_long<-reshape(data1, varying=c("ZY_1", "ZA_1", "ZL_1"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A0
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ 1,
   denominator = ~ ZL+ZY,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw1 = c(w$ipw.weights)

iptww<-iptw1; data2 <- data.frame(id,ZZY_1,ZZA_1,iptww)
#Estimate causal effect b11 (for outcome at k=1)
gee.iptw <- lm(ZZY_1~ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M11est<-out$coefficients[2,1]

data1 <- data.frame(id,ZY_2 ,ZA_2,ZL_2,ZLL_2)
data_long<-reshape(data1, varying=c("ZY_2","ZA_2","ZL_2","ZLL_2"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A1
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw2 = c(w$ipw.weights)

iptww<-iptw1*iptw2; data2 <- data.frame(id,ZZY_2,ZZA_1,ZZA_2,iptww)
#Estimate causal effects b21 and b22 (for outcome at k=2)
gee.iptw <- lm(ZZY_2~ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M21est<-out$coefficients[2,1]; M22est<-out$coefficients[3,1];


data1 <- data.frame(id,ZY_3 ,ZA_3,ZL_3,ZLL_3)
data_long<-reshape(data1, varying=c("ZY_3","ZA_3","ZL_3","ZLL_3"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A2
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw3= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3; data2 <- data.frame(id,ZZY_3, ZZA_1,ZZA_2,ZZA_3,iptww)
#Estimate causal effects b31, b32, and b33 (for outcome at k=3)
gee.iptw <- lm(ZZY_3~ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M31est<-out$coefficients[2,1]; M32est<-out$coefficients[3,1];M33est<-out$coefficients[4,1];


data1 <- data.frame(id,ZY_4 ,ZA_4,ZL_4,ZLL_4)
data_long<-reshape(data1, varying=c("ZY_4","ZA_4","ZL_4","ZLL_4"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A3
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw4= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3*iptw4; data2 <- data.frame(id,ZZY_4,ZZA_1,ZZA_2,ZZA_3,ZZA_4,iptww)
#Estimate causal effects b41, b42, b43, and b44 (for outcome at k=4)
gee.iptw <- lm(ZZY_4~ZZA_4+ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M41est<-out$coefficients[2,1]; M42est<-out$coefficients[3,1];M43est<-out$coefficients[4,1] ;M44est<-out$coefficients[5,1]

#Store estimation results
MPSI3<-c(M41est,M42est,M43est,M44est,M31est,M32est,M33est,M21est,M22est,M11est)
```

```
#Proposed method
ZY_1<-     FYwith2[,1]; ZY_2<-     FYwith2[,2]; ZY_3<-     FYwith2[,3]; ZY_4<-     FYwith2[,4]
ZL_1<-     FYwith2[,10] ;ZL_2<-     FYwith2[,11]; ZL_3<-     FYwith2[,12]; ZL_4<-     FYwith2[,13]
ZA_1<-     FYwith2[,6]; ZA_2<-     FYwith2[,7]; ZA_3<-     FYwith2[,8]; ZA_4<-    FYwith2[,9]
ZLL_2<-     FYwith2[,6]; ZLL_3<-     FYwith2[,7]; ZLL_4<-     FYwith2[,8]
ZZY_1<-     FYwith2[,2]; ZZY_2<-     FYwith2[,3]; ZZY_3<-     FYwith2[,4]; ZZY_4<-     FYwith2[,5]
ZZA_1<-     FYwith2[,6]; ZZA_2<-     FYwith2[,7]; ZZA_3<-     FYwith2[,8]; ZZA_4<-    FYwith2[,9]

id<-c(1:N);data1 <- data.frame(id,ZY_1,ZA_1,ZL_1)
data_long<-reshape(data1, varying=c("ZY_1", "ZA_1", "ZL_1"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A0
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ 1,
   denominator = ~ ZL+ZY,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw1 = c(w$ipw.weights)

iptww<-iptw1; data2 <- data.frame(id,ZZY_1,ZZA_1,iptww)
#Estimate causal effect b11 (for outcome at k=1)
gee.iptw <- lm(ZZY_1~ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M11est<-out$coefficients[2,1]

data1 <- data.frame(id,ZY_2 ,ZA_2,ZL_2,ZLL_2)
data_long<-reshape(data1, varying=c("ZY_2","ZA_2","ZL_2","ZLL_2"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A1
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw2 = c(w$ipw.weights)

iptww<-iptw1*iptw2; data2 <- data.frame(id,ZZY_2,ZZA_1,ZZA_2,iptww)
#Estimate causal effects b21 and b22 (for outcome at k=2)
gee.iptw <- lm(ZZY_2~ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M21est<-out$coefficients[2,1]; M22est<-out$coefficients[3,1];


data1 <- data.frame(id,ZY_3 ,ZA_3,ZL_3,ZLL_3)
data_long<-reshape(data1, varying=c("ZY_3","ZA_3","ZL_3","ZLL_3"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A2
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw3= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3; data2 <- data.frame(id,ZZY_3, ZZA_1,ZZA_2,ZZA_3,iptww)
#Estimate causal effects b31, b32, and b33 (for outcome at k=3)
gee.iptw <- lm(ZZY_3~ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M31est<-out$coefficients[2,1]; M32est<-out$coefficients[3,1];M33est<-out$coefficients[4,1];


data1 <- data.frame(id,ZY_4 ,ZA_4,ZL_4,ZLL_4)
data_long<-reshape(data1, varying=c("ZY_4","ZA_4","ZL_4","ZLL_4"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A3
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw4= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3*iptw4; data2 <- data.frame(id,ZZY_4,ZZA_1,ZZA_2,ZZA_3,ZZA_4,iptww)
#Estimate causal effects b41, b42, b43, and b44 (for outcome at k=4)
gee.iptw <- lm(ZZY_4~ZZA_4+ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M41est<-out$coefficients[2,1]; M42est<-out$coefficients[3,1];M43est<-out$coefficients[4,1] ;M44est<-out$coefficients[5,1]

#Store estimation results
MPSI4<-c(M41est,M42est,M43est,M44est,M31est,M32est,M33est,M21est,M22est,M11est)
```

```
#(Univariate) correlation-preserving factor score centering condition
ZY_1<-     (YY0-mean(YY0)-FYres)
ZY_2<-     (YY1-mean(YY1)-FYres)
ZY_3<-     (YY2-mean(YY2)-FYres)
ZY_4<-     (YY3-mean(YY3)-FYres)
ZA_1<-AA0-mean(AA0)-FAres; ZA_2<-AA1-mean(AA1)-FAres; ZA_3<- AA2-mean(AA2)-FAres; ZA_4<- AA3-mean(AA3)-FAres;
ZL_1<- LL0-mean(LL0)-FLres; ZL_2<- LL1-mean(LL1)-FLres; ZL_3<- LL2-mean(LL2)-FLres; ZL_4<- LL3-mean(LL3)-FLres
ZLL_2<- AA0-mean(AA0)-FAres; ZLL_3<- AA1-mean(AA1)-FAres; ZLL_4<- AA2-mean(AA2)-FAres
ZZY_1<-     (YY1-mean(YY1)-FYres)
ZZY_2<-     (YY2-mean(YY2)-FYres)
ZZY_3<-     (YY3-mean(YY3)-FYres)
ZZY_4<-     (YY4-mean(YY4)-FYres)
ZZA_1<- AA0-mean(AA0)-FAres; ZZA_2<- AA1-mean(AA1)-FAres; ZZA_3<- AA2-mean(AA2)-FAres; ZZA_4<- AA3-mean(AA3)-FAres;

id<-c(1:N);data1 <- data.frame(id,ZY_1,ZA_1,ZL_1)
data_long<-reshape(data1, varying=c("ZY_1", "ZA_1", "ZL_1"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A0
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ 1,
   denominator = ~ ZL+ZY,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw1 = c(w$ipw.weights)

iptww<-iptw1; data2 <- data.frame(id,ZZY_1,ZZA_1,iptww)
#Estimate causal effect b11 (for outcome at k=1)
gee.iptw <- lm(ZZY_1~ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M11est<-out$coefficients[2,1]

data1 <- data.frame(id,ZY_2 ,ZA_2,ZL_2,ZLL_2)
data_long<-reshape(data1, varying=c("ZY_2","ZA_2","ZL_2","ZLL_2"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A1
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw2 = c(w$ipw.weights)

iptww<-iptw1*iptw2; data2 <- data.frame(id,ZZY_2,ZZA_1,ZZA_2,iptww)
#Estimate causal effects b21 and b22 (for outcome at k=2)
gee.iptw <- lm(ZZY_2~ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M21est<-out$coefficients[2,1]; M22est<-out$coefficients[3,1];

data1 <- data.frame(id,ZY_3 ,ZA_3,ZL_3,ZLL_3)
data_long<-reshape(data1, varying=c("ZY_3","ZA_3","ZL_3","ZLL_3"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A2
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw3= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3; data2 <- data.frame(id,ZZY_3, ZZA_1,ZZA_2,ZZA_3,iptww)
#Estimate causal effects b31, b32, and b33 (for outcome at k=3)
gee.iptw <- lm(ZZY_3~ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M31est<-out$coefficients[2,1]; M32est<-out$coefficients[3,1];M33est<-out$coefficients[4,1];

data1 <- data.frame(id,ZY_4 ,ZA_4,ZL_4,ZLL_4)
data_long<-reshape(data1, varying=c("ZY_4","ZA_4","ZL_4","ZLL_4"), direction="long", idvar="id", sep="_")
data_long_sort <- arrange(data_long, id, time)

#Calculate weight for A3
w <- ipwtm(exposure = ZA,family = "gaussian",link = "logit",numerator = ~ ZLL,
   denominator = ~ ZL+ZY+ZLL,
   id = id,timevar=time,corstr="ar1", type="all",data = data_long_sort)
iptw4= c(w$ipw.weights)

iptww<- iptw1*iptw2*iptw3*iptw4; data2 <- data.frame(id,ZZY_4,ZZA_1,ZZA_2,ZZA_3,ZZA_4,iptww)
#Estimate causal effects b41, b42, b43, and b44 (for outcome at k=4)
gee.iptw <- lm(ZZY_4~ZZA_4+ZZA_3+ZZA_2+ZZA_1,data=data2, weights=iptww)
out<-summary(gee.iptw) ; M41est<-out$coefficients[2,1]; M42est<-out$coefficients[3,1];M43est<-out$coefficients[4,1] ;M44est<-out$coefficients[5,1]

#Store estimation results
MPSI5<-c(M41est,M42est,M43est,M44est,M31est,M32est,M33est,M21est,M22est,M11est)
```

```
ITER<-ifelse(ITER1==200,0,1)+ ifelse(ITER2==200,0,1)+ ifelse(ITER3==200,0,1)+ ifelse(ITER5==200,0,1)
IMPMSM<-ifelse(is.na(prod(MPSI1,MPSI2,MPSI3,MPSI4,MPSI5)),0,1)

if(IMPRO1+IMPRO2+IMPRO3+IMPRO4+ITER+IMPMSM>26){
#Y4A3 0.4
#Y4Y3A2 0.4*0.4#Y4L3A2 0.1*0.2
#Y4Y3Y2A1 0.4*0.4*0.4#Y4Y3L2A1 0.4*0.1*0.2#Y4L3Y2A1 0.1*0.2*0.4#Y4L3L2A1 0.1*0.5*0.2
#Y4Y3Y2Y1A0 0.4*0.4*0.4*0.4#Y4Y3Y2L1A0 0.4*0.4*0.1*0.2 #Y4Y3L2Y1A0 0.4*0.1*0.2*0.4 #Y4Y3L2L1A0 0.4*0.1*0.5*0.2
#Y4L3Y2Y1A0 0.1*0.2*0.4*0.4#Y4L3Y2L1A0 0.1*0.2*0.1*0.2
#Y4L3L2Y1A0 0.1*0.5*0.2*0.4#Y4L3L2L1A0 0.1*0.5*0.5*0.2
#True values of causal effects
PTRUE<-c(0.400,0.180,0.090,0.0486,0.400,0.180,0.090,0.400,0.180,0.400)

#Store point estimates results
POINTEST[3*(aaaa-1)+dddd,1]<- POINTEST[3*(aaaa-1)+dddd,1]+4
POINTEST[3*(aaaa-1)+dddd,2]<- POINTEST[3*(aaaa-1)+dddd,2]+aaaa
POINTEST[3*(aaaa-1)+dddd,3]<- POINTEST[3*(aaaa-1)+dddd,3]+bbbb
POINTEST[3*(aaaa-1)+dddd,4]<- POINTEST[3*(aaaa-1)+dddd,4]+cccc
POINTEST[3*(aaaa-1)+dddd,5]<- POINTEST[3*(aaaa-1)+dddd,5]+dddd
POINTEST[3*(aaaa-1)+dddd,6:15]<- POINTEST[3*(aaaa-1)+dddd,6:15]+PSI1 -PTRUE
POINTEST[3*(aaaa-1)+dddd,16:25]<- POINTEST[3*(aaaa-1)+dddd,16:25]+ PSI2 -PTRUE
POINTEST[3*(aaaa-1)+dddd,26:35]<- POINTEST[3*(aaaa-1)+dddd,26:35]+ PSI3 -PTRUE
POINTEST[3*(aaaa-1)+dddd,36:45]<- POINTEST[3*(aaaa-1)+dddd,36:45]+ PSI4 -PTRUE
POINTEST[3*(aaaa-1)+dddd,46:55]<- POINTEST[3*(aaaa-1)+dddd,46:55]+ PSI5 -PTRUE
POINTEST[3*(aaaa-1)+dddd,56:65]<- POINTEST[3*(aaaa-1)+dddd,56:65]+ MPSI1 -PTRUE
POINTEST[3*(aaaa-1)+dddd,66:75]<- POINTEST[3*(aaaa-1)+dddd,66:75]+ MPSI2 -PTRUE
POINTEST[3*(aaaa-1)+dddd,76:85]<- POINTEST[3*(aaaa-1)+dddd,76:85]+ MPSI3 -PTRUE
POINTEST[3*(aaaa-1)+dddd,86:95]<- POINTEST[3*(aaaa-1)+dddd,86:95]+ MPSI4 -PTRUE
POINTEST[3*(aaaa-1)+dddd,96:105]<- POINTEST[3*(aaaa-1)+dddd,96:105]+ MPSI5 -PTRUE

#Store SE estimates results
SEEST[3*(aaaa-1)+dddd,1]<- SEEST[3*(aaaa-1)+dddd,1]+4
SEEST[3*(aaaa-1)+dddd,2]<- SEEST[3*(aaaa-1)+dddd,2]+aaaa
SEEST[3*(aaaa-1)+dddd,3]<- SEEST[3*(aaaa-1)+dddd,3]+bbbb
SEEST[3*(aaaa-1)+dddd,4]<- SEEST[3*(aaaa-1)+dddd,4]+cccc
SEEST[3*(aaaa-1)+dddd,5]<- SEEST[3*(aaaa-1)+dddd,5]+dddd
SEEST[3*(aaaa-1)+dddd,6:15]<- SEEST[3*(aaaa-1)+dddd,6:15]+(PSI1 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,16:25]<- SEEST[3*(aaaa-1)+dddd,16:25]+ (PSI2 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,26:35]<- SEEST[3*(aaaa-1)+dddd,26:35]+ (PSI3 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,36:45]<- SEEST[3*(aaaa-1)+dddd,36:45]+ (PSI4 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,46:55]<- SEEST[3*(aaaa-1)+dddd,46:55]+ (PSI5 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,56:65]<- SEEST[3*(aaaa-1)+dddd,56:65]+ (MPSI1 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,66:75]<- SEEST[3*(aaaa-1)+dddd,66:75]+ (MPSI2 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,76:85]<- SEEST[3*(aaaa-1)+dddd,76:85]+ (MPSI3 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,86:95]<- SEEST[3*(aaaa-1)+dddd,86:95]+ (MPSI4 -PTRUE)^2
SEEST[3*(aaaa-1)+dddd,96:105]<- SEEST[3*(aaaa-1)+dddd,96:105]+ (MPSI5 -PTRUE)^2

COUNT<-COUNT+1
}else{;}
}else{;}

}
;};}

KKKK1<-read.csv("*****/pointest41.csv")[,-1]
if(KKKK1[1,5]>199){;q();}else{;}
KKKK1[,106]<-KKKK1[,106]+1
POINTEST<-POINTEST+KKKK1
write.csv(POINTEST, "*****/pointest41.csv")

KKKK2<-read.csv("*****/seest41.csv")[,-1]
KKKK2[,106]<-KKKK2[,106]+1
SEEST<-SEEST+KKKK2
#Save simulation results in directory
write.csv(SEEST, "*****/seest41.csv")
}
```